Thermo-Fluidic Processes in Spatially Interconnected Structures

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Where innovation starts

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Thermo-Fluidic Processes

Mutual effect of thermal energy on interacting solids and fluids.

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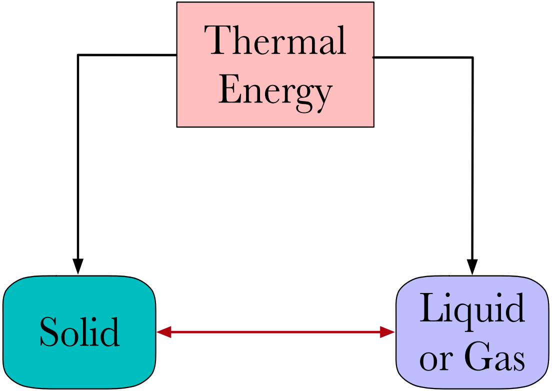
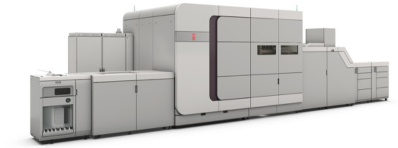


Thermo-Fluidic Processes: Examples



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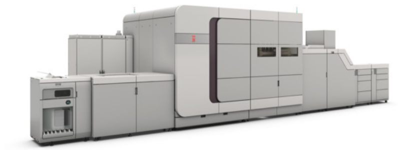

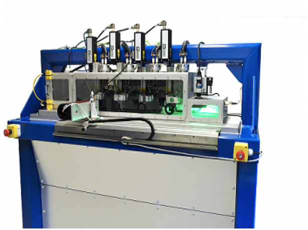

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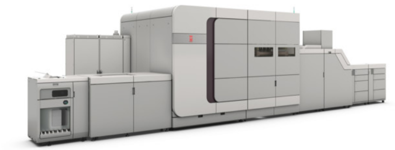

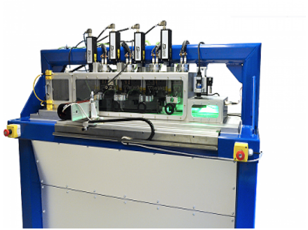

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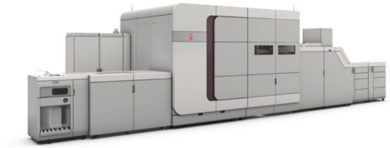

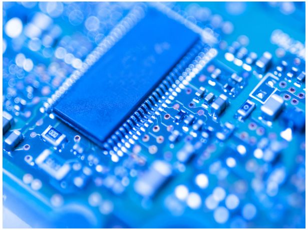

Thermo-Fluidic Processes in Inkjet Printing

Inkjet printing is a physical integration of **liquid material** and **solid medium**.

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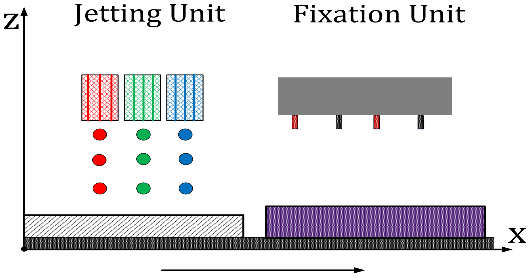

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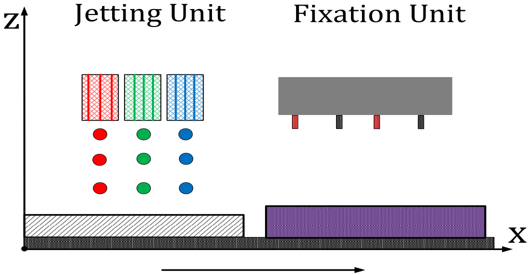
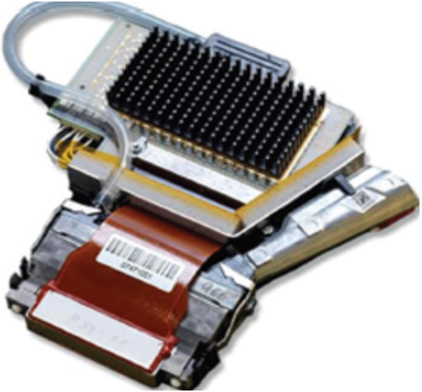


Thermo-fluidic processes affect the Print Quality.

How Do Thermo-Fluidic Processes Influence Print Quality?

Thermo-fluidic processes in Jetting.



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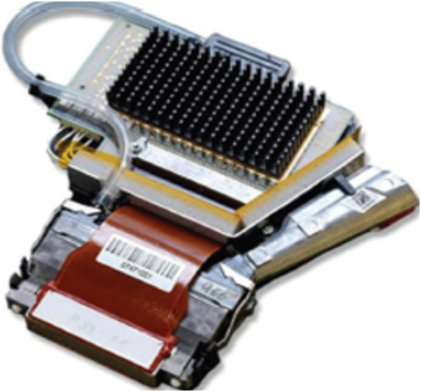

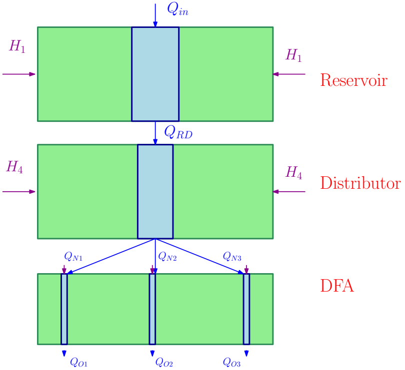

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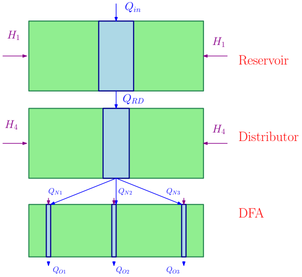

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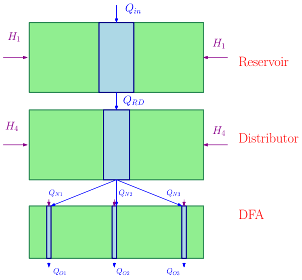

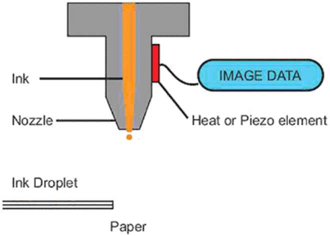

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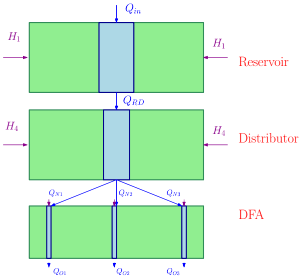
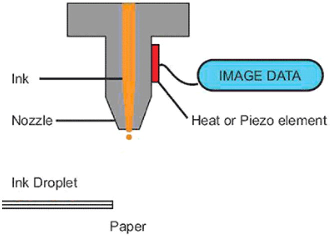


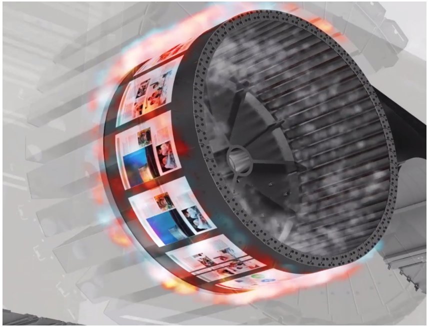


What do we want?

Every liquid droplet from individual nozzle should maintain a desired temperature.

How Do Thermo-Fluidic Processes Influence Print Quality?

Thermo-fluidic processes in Fixation.



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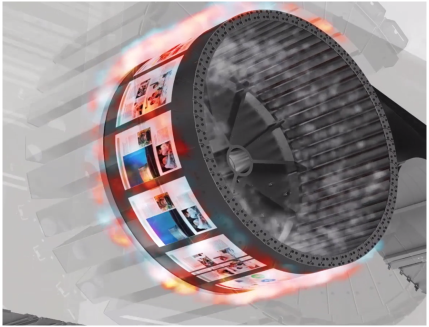

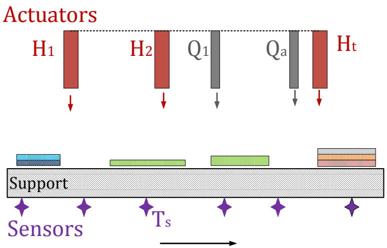

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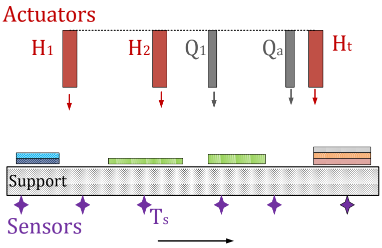

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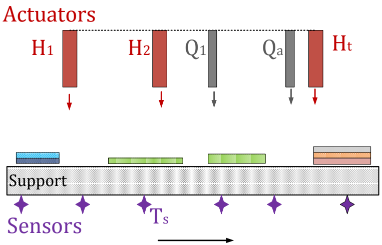

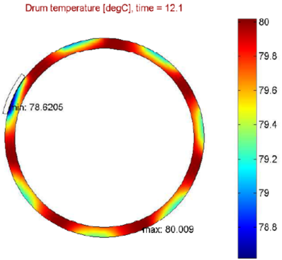

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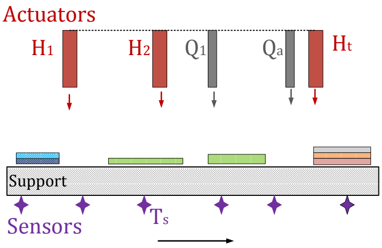
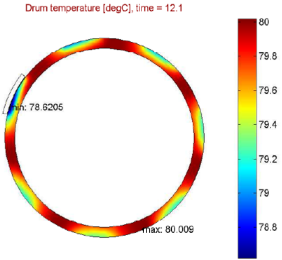
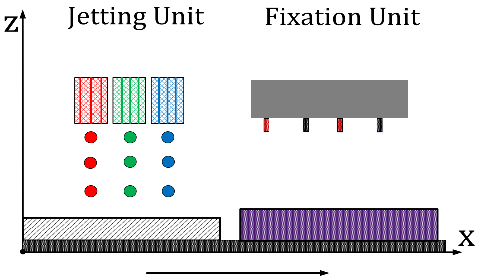




What do we want?

Individual sheet of paper should have a desired temperature & moisture distribution.

Thermo-Fluidic Processes in Inkjet Printing



Common Properties of Thermo-Fluidic Processes:

1. Coupled Multi-variable PDEs.
2. Energy exchange of interacting physical phenomena over boundaries.

A Bigger Picture

How to fully exploit the mutual interaction among different physical phenomena for solving model-based problems that are governed by PDEs?

Specifically:

1. How do we represent the interconnection?
2. How to quantify 'energy' dissipation at the interconnection?
3. Can we always construct or deconstruct these systems in components that are meaningful?

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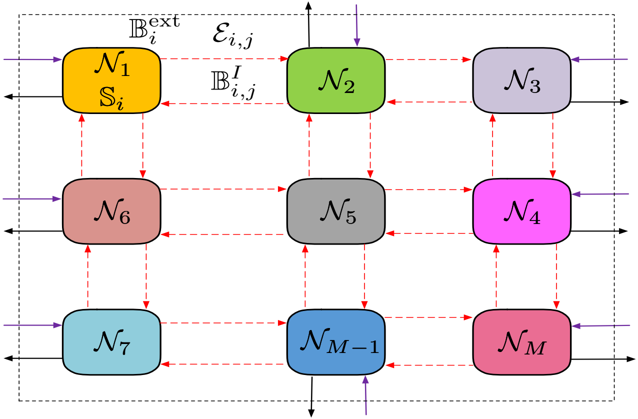
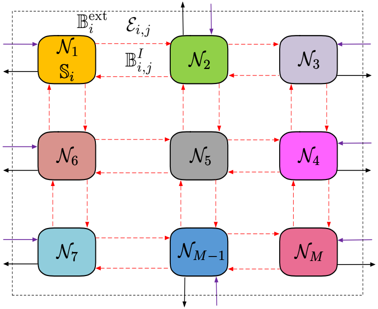
Modeling Thermo-Fluidic Processes: Graph Theoretic Framework

A dynamic network of infinite dimensional systems.



Modeling Thermo-Fluidic Processes: Topology Description

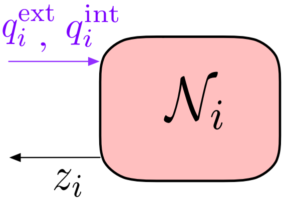


Topology

1. A finite connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$
2. An adjacency matrix A.

- $\mathcal{N}_i \in \mathcal{N}$ denotes infinite dimensional dynamics of individual node.
- $\mathcal{E}_{i,j} \in \mathcal{E}$ denotes the interconnection of adjacent nodes.

Modeling Thermo-Fluidic Processes: Governing Dynamics

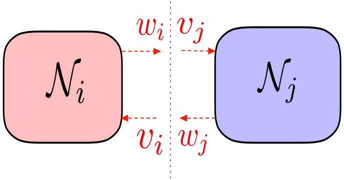


1. The state variables: $z_i \colon \mathbb{S}_i \times \mathbb{T} \to \mathbb{R}^{n_i}, \quad \mathbb{S}_i \subseteq \mathbb{R}^3$.
2. The boundary inputs: $q_i^{\text{ext}} \colon \mathbb{B}_i^{\text{ext}} \times \mathbb{T} \to \mathbb{R}^{n_i}, \quad \mathbb{B}_i^{\text{ext}} \subseteq \mathbb{R}^2$.
3. The in-domain inputs: $q_i^{\text{int}} \colon \mathbb{B}_i^{\text{int}} \times \mathbb{T} \to \mathbb{R}^{m_i}, \quad \mathbb{B}_i^{\text{int}} \subseteq \mathbb{S}_i$.

$$\text{PDE Model}(\mathrm{D}_i) \begin{cases} \mathcal{E}_i \frac{\partial z_i}{\partial t} = \mathcal{A}_i z_i + \mathcal{B}_i^{\text{int}} q_i^{\text{int}} \\ \mathcal{A}_i z_i := \boldsymbol{\nabla} \cdot [K_i(s)\boldsymbol{\nabla} - \boldsymbol{V}_i(s)] z_i \end{cases}$$

$$\text{External boundaries } (\mathrm{B}_i^{\text{ext}}) \begin{cases} \mathcal{H}_i^{\text{ext}} \ z_i = q_i^{\text{ext}}. \\ \mathcal{H}_i^{\text{ext}} \ z_i := [K_i(s) \frac{\partial}{\partial \boldsymbol{b}_i^{\text{ext}}} - \boldsymbol{V}_i \cdot \boldsymbol{b}_i^{\text{ext}} + H_i^{\text{ext}}(s)] z_i \end{cases}$$

Modeling Thermo-Fluidic Processes: Governing Dynamics



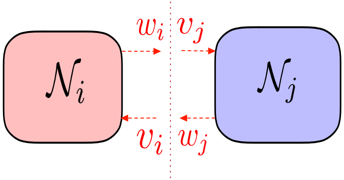
1. Inputs: $v_i := [K_i(s)\frac{\partial}{\partial \boldsymbol{b}^l_{i,j}} - \boldsymbol{V}_i \cdot \boldsymbol{b}^l_{i,j}]z_i, \quad s \in \mathbb{B}^l_{i,j}$.
2. Outputs: $w_i := z_i, \quad s \in \mathbb{B}^l_{i,j}$.

1. Storage Function $V_i := \left\langle \mathcal{E}_i z_i, z_i \right\rangle_{L_2(\mathbb{S}_i)}$
2. Supply Function $S^l_{i,j} := \left\langle w_i, v_i \right\rangle_{L_2(\mathbb{B}^l_{i,j})}$

Edge $\mathcal{E}_{i,j}$ as a dissipative interconnection ($\mathrm{B}^l_{i,j}$):

1. **Loss-less edge:** $\mathrm{B}^l_{i,j} := S_{i,j} + S_{j,i} = 0$.
2. **Lossy edge:** $\mathrm{B}^l_{i,j} := S_{i,j} + S_{j,i} < 0$.

Modeling Thermo-Fluidic Processes: Governing Dynamics



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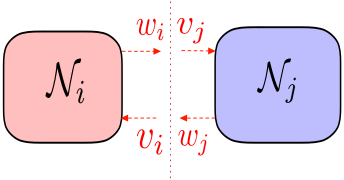
1. $\frac{\mathrm{d}V_i}{\mathrm{d}t} \leq S^l_{i,j}$: **every autonomous and regular interconnectant is dissipative**.
2. $\sum_{i=1}^{M} \frac{\mathrm{d}V_i}{\mathrm{d}t} \leq 0$: **interconnection of dissipative components is stable and dissipative**.

Modeling Thermo-Fluidic Processes: Governing Dynamics



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Open Questions:

1. **Thermo-dynamic interpretation of the edges.**
2. **Model reduction of individual node without disrupting the edge behavior.**

Modeling Thermo-Fluidic Processes: Summary

A well-posed dynamic network of infinite dimensional models.

1. A finite and connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$.
2. An adjacency matrix A.
3. Every node $\mathcal{N}_i$ describes the thermal-fluidic process of a component

$$\mathcal{N}_i = (\mathbb{S}_i, \mathbb{B}_i^{\text{ext}}, \mathbb{B}_i^{\text{int}}, \mathsf{D}_i, \mathsf{B}_i^{\text{ext}}).$$

4. Every edge $\mathcal{E}_{i,j}$ describes the interconnection of adjacent thermo- fluidic processes

$$\mathcal{E}_{i,j} = (\mathbb{B}_{i,j}^{I}, \mathsf{B}_{i,j}^{I}).$$

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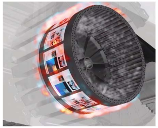
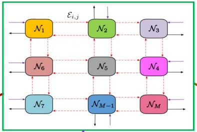
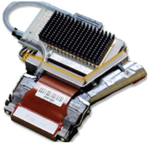
$$\mathcal{E}_{i,j} = (\mathbb{B}_{i,j}^{I}, \mathrm{B}_{i,j}^{I}).$$

Still the model is infinite dimensional!

Three-ways to Solve A Model-Based Problem





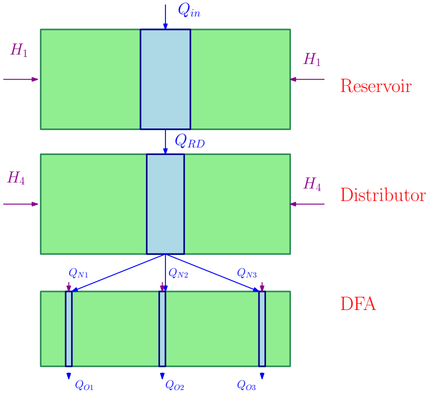


Infinite Dimensional Controller/Estimator → (Discretization) → Finite Dimensional Controller/Estimator → Reduced Order Controller/ Estimator → Implementation

(Lumping) → Lumped ODE (NL) model replacing PDEs → Finite Dimensional Controller/Estimator → Reduced Order Controller/ Estimator → Implementation

(Discretization) → Finite Dimensional Large Scale DAE structure → Reduced Order Problem → Finite Dimensional Controller/ Estimator → Implementation

Option 1: Lumped Model-Based Control of Jetting Process



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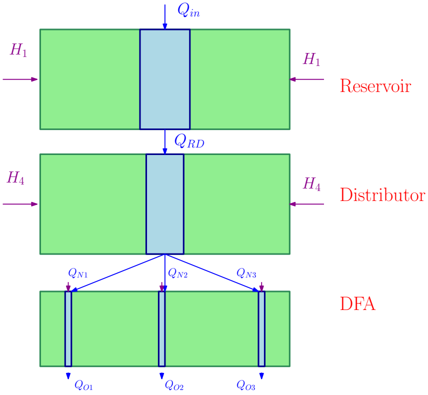

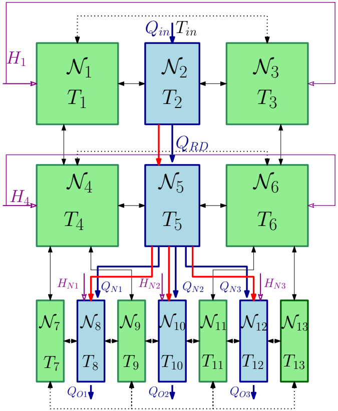

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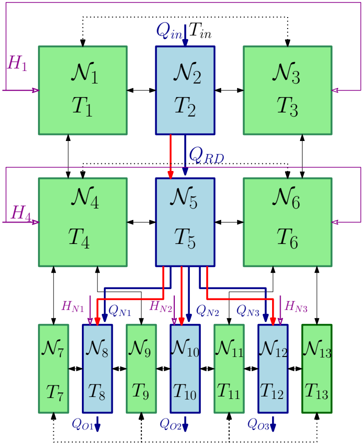


Network of finite dimensional systems

1. A finite connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$
2. An adjacency matrix $A \neq A^\top$
3. Set of edges $\mathcal{E} = \{\mathcal{E}_{i,j} \mid \text{for all } (i,j) \text{ with } A_{i,j} = 1\}$
4. Set of nodes $\mathcal{N} = \{\mathcal{N}_i := \mathcal{P}_i | i = 1, \cdots, L\}$

$$\mathcal{P}_i := \left\{ \begin{bmatrix} \dot{x}_i(t) \\ w_i(t) \\ y_i(t) \end{bmatrix} = \begin{bmatrix} A^i_{\theta xx} & A^i_{\theta xv} & B^i_{\theta xu} \\ A^i_{\theta wx} & A^i_{\theta wv} & B^i_{\theta wu} \\ C^i_{\theta yx} & C^i_{\theta yv} & D^i_{\theta yu} \end{bmatrix} \begin{bmatrix} x_i(t) \\ v_i(t) \\ u_i(t) \end{bmatrix} \right.$$

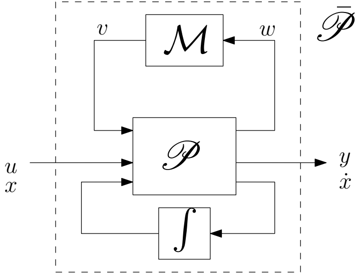
$$\mathcal{E}_{i,j} := \{w_{i,j} = v_{j,i} | A_{i,j} = 1, i \neq j\}.$$

Option 1: Lumped Model-Based Control of Jetting Process

$$\mathcal{P} := \left\{ \begin{bmatrix} \dot{x}(t) \\ w(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \mathrm{diag} A_{\theta xx} & \mathrm{diag} A_{\theta xv} & \mathrm{diag} B_{\theta xu} \\ \mathrm{diag} A_{\theta wx} & \mathrm{diag} A_{\theta wv} & \mathrm{diag} B_{\theta wu} \\ \mathrm{diag} C_{\theta yx} & \mathrm{diag} C_{\theta yv} & \mathrm{diag} D_{\theta yu} \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \\ u(t) \end{bmatrix} \right.$$

Interconnection Matrix:

$$v = \mathcal{M} w$$



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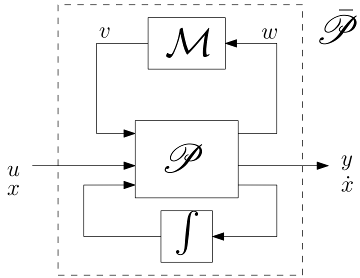

LFR with $\mathcal{M}$ in Feedback

1. Interconnections are well-posed iff $\operatorname{diag}(I - A_{\theta wv}\mathcal{M}) \neq 0$.
2. We can obtain a full-block LPV model from the LFR structure.

Centralized Output Tracking MPC on Jetting Process: Simulation

Feature of the controller

Anticipation and **pre-compensation** of the temperature changes

Controller's Job

1. Utilise prior information of print-profile
2. Respect the constraints on states and inputs

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Centralized Output Tracking MPC on Jetting Process: Simulation

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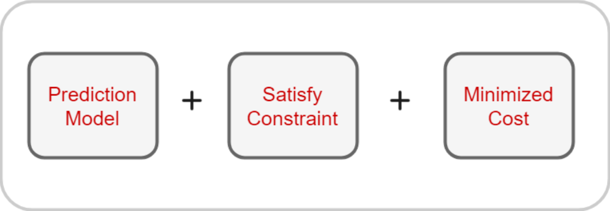

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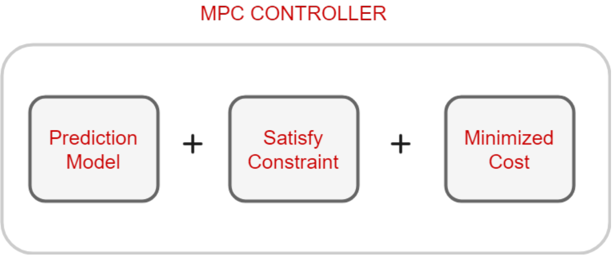

Lumped Model-Based Control of Jetting Process: Centralized Control

What do we want?

$$|T_i(t) - T_{ref}| \leq 0.01^\circ C, \quad \forall t \geq T \quad \text{and} \quad \forall i = 1, \cdots, N_m$$

MIMO State Space Model:

$$\bar{\mathcal{P}} := \begin{cases} \dot{x}(t) & = A(\theta(t))x(t) + B(\theta(t))u(t) + f(\theta(t)) \\ y(t) & = Cx(t) \end{cases}$$

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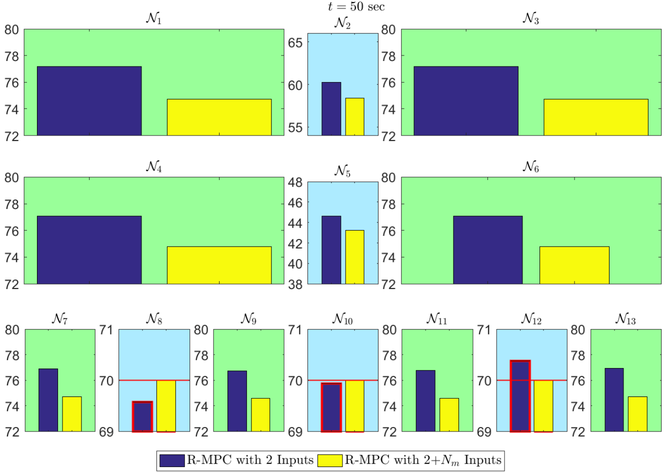
Two actuation scenarios

1. Only using the 2 heating inputs in solid blocks.
2. Using additional N_m piezo electric actuators as heating inputs.

Centralized Output Tracking MPC on Jetting Process: Simulation

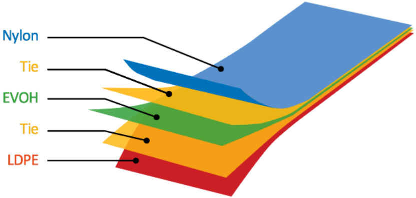


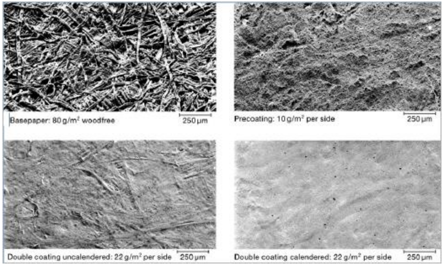
Lumped Model-Based Control of Jetting Process: Future Work

1. **Distributed Optimization:** Decomposing the centralized problems into sub-problems that can be solved in parallel and/or sequential manner.
2. **Soft-sensing Temperature:** Using self-sensing mechanism of piezo-electric actuators.
3. **Design of Actuation Pulse:** Thermal actuation of the piezo-electric actuators should not form drops.

Option 2: Parameter Estimation of Paper-Sheet in Fixation

Modeling the fixation process requires physical parameters of the papers.





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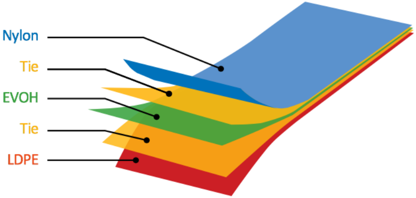
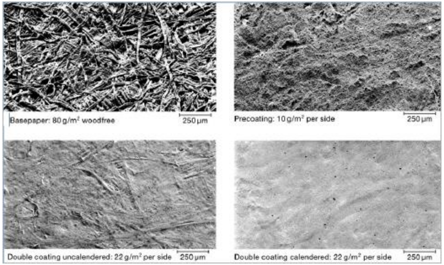
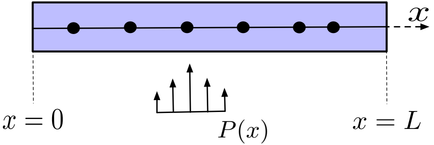




Input-output based estimation of Spatially Varying physical coefficients

Parameter Estimation of Paper-Sheet in Fixation in Frequency Domain



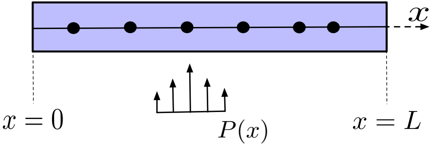
PDE: $E(x)\frac{\partial z}{\partial t} = D(x)\frac{\partial^2 z}{\partial x^2} + U(x)\frac{\partial z}{\partial x} + K(x)z + P(x)q(t).$

Measured Output: $y^m(\omega_\ell) = \mathrm{col}(z(x^1,\omega_\ell),\cdots,z(x^M,\omega_\ell)), \omega_\ell \in \mathbb{W}.$

Estimation Problem: $\min\limits_{\theta:=\mathrm{col}(E,D,U,K,P)} \sum_{\ell=1}^{L} \int_{\mathbb{X}^M} \| \, y^m(\omega_\ell) - H(\omega_\ell,x,\theta) \; q(\omega_\ell) \, \|^2 \, dx.$

- Boundary conditions are unknown.

Parameter Estimation of Paper-Sheet in Fixation in Frequency Domain



Parametrization of Spatially Varying Functions: $\gamma(x) \approx \sum_{r=1}^{R} B_r(x)\theta_r.$

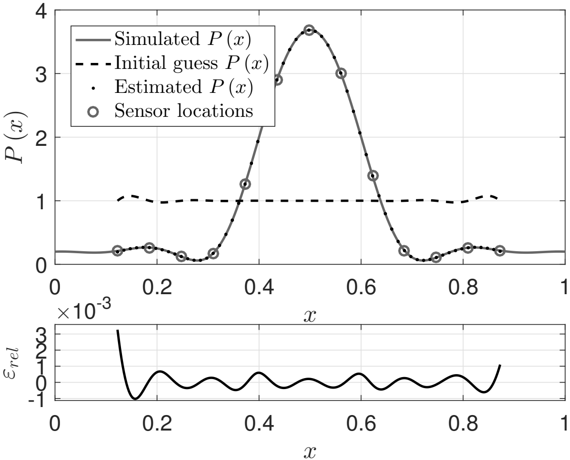
Finite Difference/Volume Discretization of PDEs:
$$\begin{aligned} \dot{\mathbf{z}} &= A(\bar{\theta})\mathbf{z} + B(\bar{\theta})q, \\ y_m &= C^m \mathbf{z}. \end{aligned}$$

Estimation Problem: $\min_{\bar{\theta}} \sum_{\ell=1}^{L} \sum_{m=2}^{M-1} \frac{1}{w^m w_\ell} \mid y^m(\omega_\ell, x^m) - G^m(j\omega_\ell, \bar{\theta})\, q(\omega_\ell) \mid^2 .$

- Two extremum measurements are considered as inputs to mimic boundaries.

Simulation Results: $\frac{\partial z}{\partial t} = D(x)\frac{\partial^2 z}{\partial x^2} + P(x)q(t)$.

The basis functions $B_r(x)$ are known a priori.



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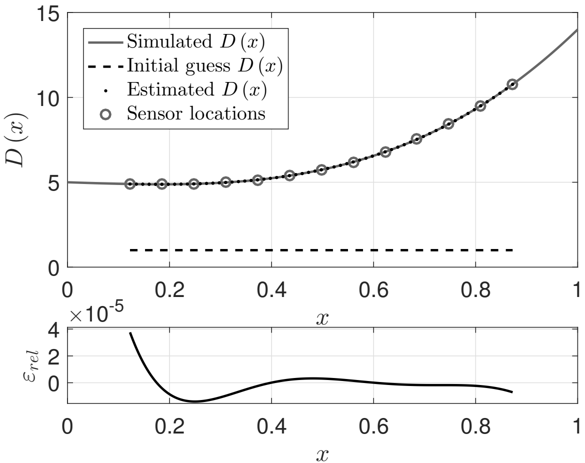

Parameter Estimation of Paper Sheet: Open Problems

Currently we are implementing it for estimating parameters of papers in an experimental set-up.

1. **What if the basis functions are unknown?**
2. **Optimal experiment design?**
3. **Convexifying the optimization problem?**
4. **Any alternative/ innovative approach?**

Conclusions

General Conclusions

1. Interconnection among thermo-fluidic processes can be viewed as dissipative exchange of energy.
2. Finite dimensional lumping of infinite dimensional models poses similar interconnection structure.
3. Infinite dimensional models require discretization, however, the stage at which the discretization has to be performed is not trivial.

Conclusions

Application Specific Conclusions

1. Piezo electric actuators as additional control inputs improve the the performance in jetting process.
2. Utilizing the a priori knowledge of the flow pattern per nozzle can improve the performance of the jetting process.
3. Physical parameters of papers typically vary over spatial domain that are estimated using spatially distributed sensor array.

Thank You!