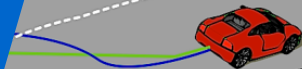


# Optimal Trajectory Tracking Control for Automated Guided Vehicles

Amritam Das, Yanin Kasemsinsup, Siep Weiland

**Control Systems Group**

Dept. of Electrical Engineering



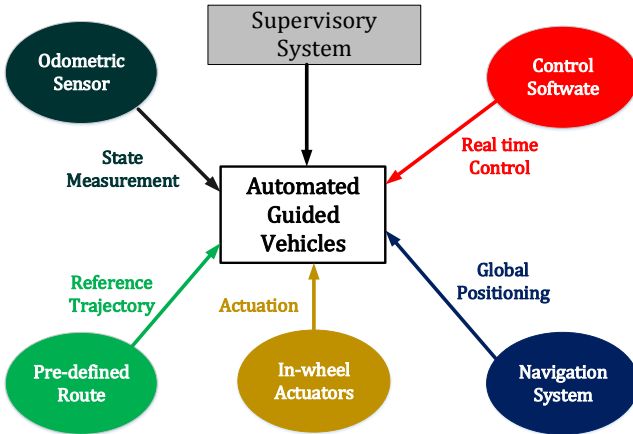
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# Automated Guided Vehicles (AGVs)

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**Supervised autonomous driving in pre-defined route.**



# Automated Guided Vehicles (AGVs)

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# Automated Guided Vehicles (AGVs)

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**Different application, identical driving principal.**



# Research Question

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**How to develop a control strategy for automated guided vehicles which tracks a pre-defined trajectory ?**

Features:

1. Generic for any kind of AGV with arbitrary number of wheels.
2. Handling severe cornering maneuver.
3. Carrying heavy load in elevated or banked road surface.



# Research Question

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**How to develop a control strategy for automated guided vehicles which tracks a pre-defined trajectory ?**

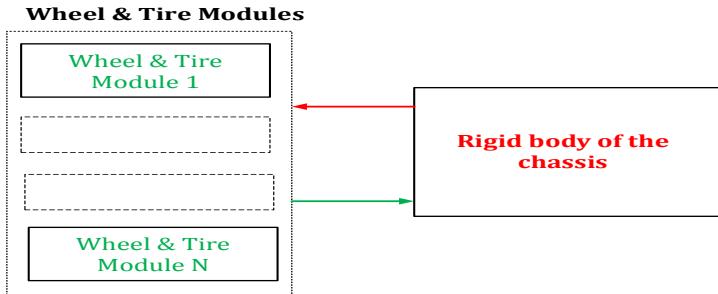
Features:

1. Generic for any kind of AGV with arbitrary number of wheels.
2. Handling severe cornering maneuver.
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# System Overview

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**Vehicle as multibody system.**



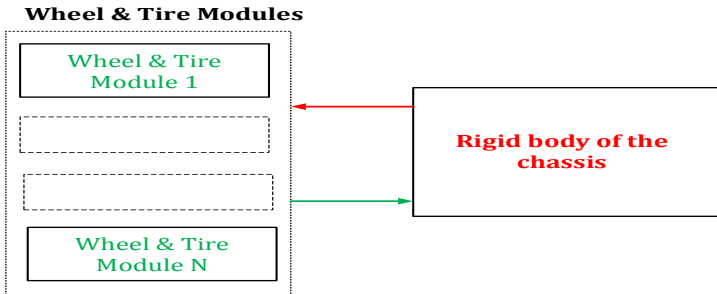
## Observation

Separate the control problem of each wheel & tire module from chassis.

# System Overview

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**Vehicle as multibody system.**

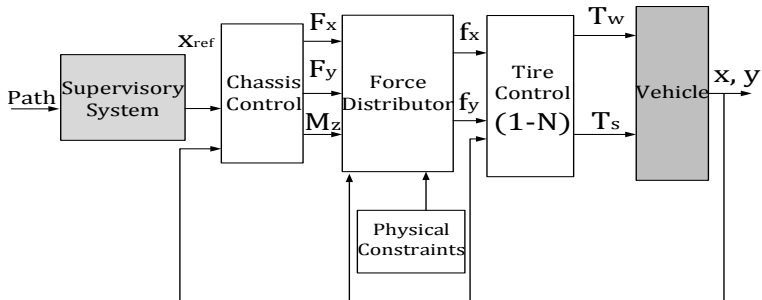


## Observation

Separate the control problem of each wheel & tire module from chassis.

# Controller Architecture

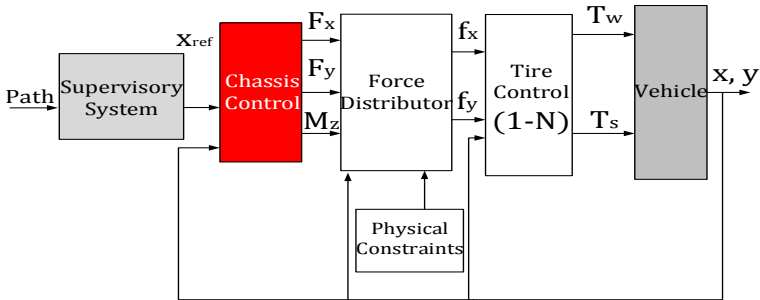
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Cascade Control structure.

# Controller Architecture

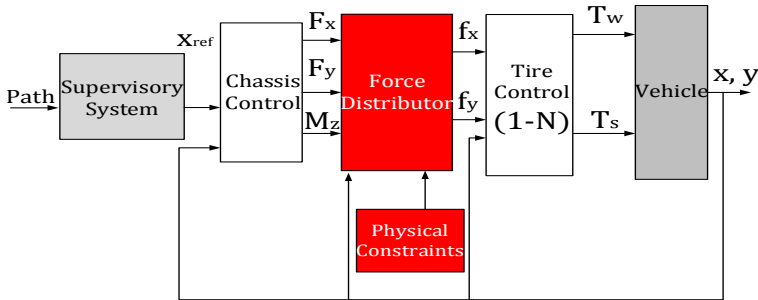
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**Determine the optimal longitudinal, lateral body force and also the yaw moment to be applied to the center of mass of the chassis.**

# Controller Architecture

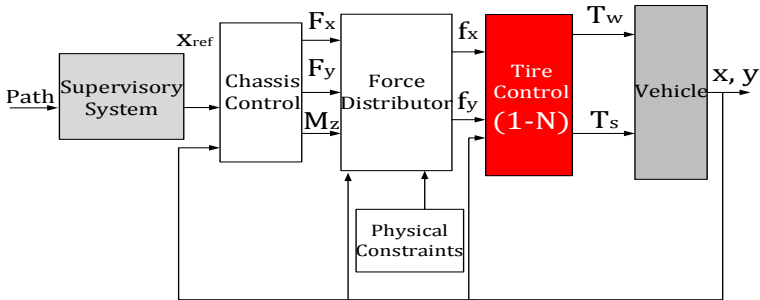
5 / 12



**Distribute the desired forces and moment from the chassis controller over N controllable wheels, under physical constraints.**

# Controller Architecture

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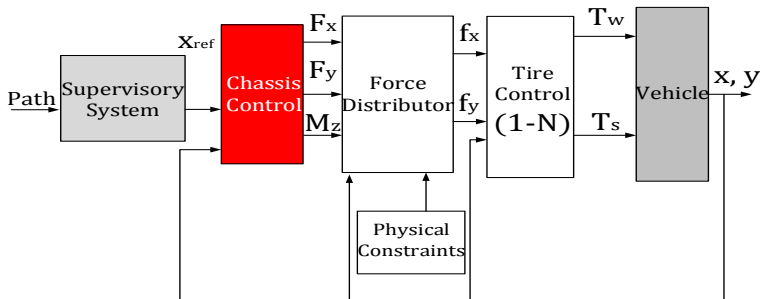
**Determine the control input for each in-wheel actuator to track desired wheel-forces.**

# Chassis Control

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## Objective

Determine the desired  $u_b := [F_x \ F_y \ M_z]^T$  for given  $x_{\text{ref}}(t)$ .



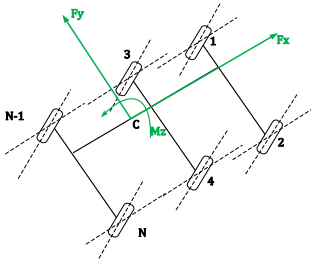


# Chassis Control

6 / 12

## Objective

Determine the desired  $u_b := [F_x \ F_y \ M_z]^T$  for given  $x_{\text{ref}}(t)$ .



## Considerations:

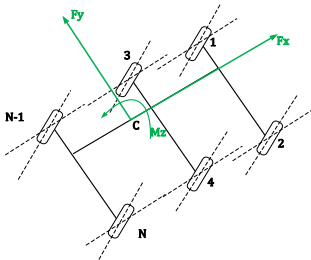
- ▶ Chassis as rigid body.
- ▶ Including load, new center of mass is calculated.
- ▶ Nonlinear dynamics of the chassis  $\dot{x}_b = f_b(x_b, u_b)$ .
- ▶  $x_b$  includes **longitudinal velocity, lateral velocity, yaw rate, roll, roll rate.**

# Chassis Control

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## Objective

Determine the desired  $u_b := [F_x \ F_y \ M_z]^T$  for given  $x_{\text{ref}}(t)$ .



## Steps:

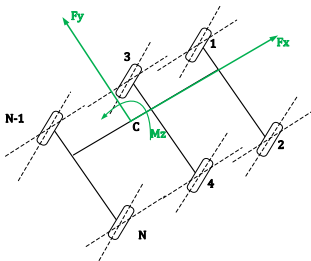
- ▶ Divide  $x_{\text{ref}}(t)$  into finite segments.
- ▶ Linearize the model for each segment.
- ▶ Apply receding horizon LQ optimal control.

# Chassis Control

6 / 12

## Objective

Determine the desired  $F_x$ ,  $F_y$  and  $M_z$  for  $x_{\text{ref}}(t)$  with  $t \in [t_k, t_{k+1}]$ .



## Steps:

- ▶ Divide  $x_{\text{ref}}(t)$  into finite segments.
- ▶ Linearize the model for each segment.
- ▶ Apply receding horizon LQ optimal control.

## Cost Functional for $k^{\text{th}}$ segment:

$$J(x_b^*, x_{\text{ref}}, u_b) = e^T(t_{k+1}) Q_f e(t_{k+1}) + \int_{t_k}^{t_{k+1}} [e^T(t) Q e(t) + u_b^T(t) R u_b(t)] dt$$

Tracking error  $e(t) := x_{\text{ref}}(t) - x_b(t)$

# Chassis Control

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ARE based State Feedback:

$$A^T K + K A - K B R^{-1} B^T K + Q = 0,$$

# Chassis Control

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Anticipative Feedforward:

$$\dot{r}_b(t) = -[A^T - K B R^{-1} B^T] r_b(t) + Q x_{\text{ref}}(t),$$

$$r_b(t_{k+1}) = -Q_f x_{\text{ref}}(t_{k+1})$$

# Chassis Control

6 / 12

ARE based State Feedback:

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Control Input:

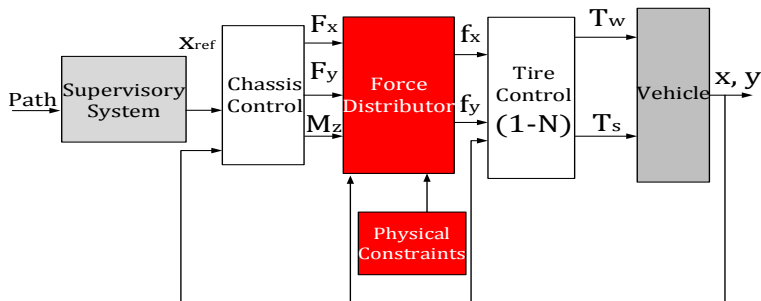
$$u_{b,\text{opt}}(t) = -R^{-1} B^T [K x_b(t) + r_b(t)].$$

# Force Distributor

7 / 12

## Objective

Distribute desired  $F_x$ ,  $F_y$  and  $M_z$  to each wheel tire module.

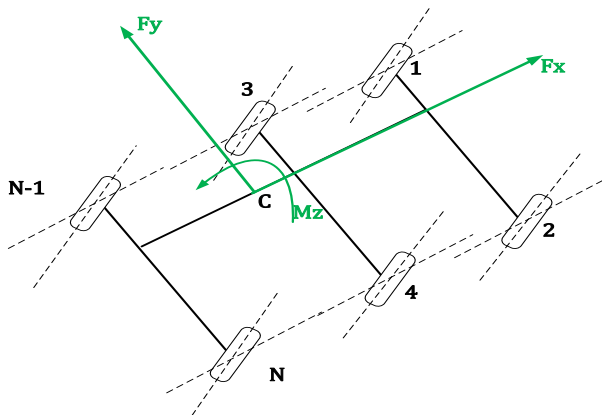


# Force Distributor

7 / 12

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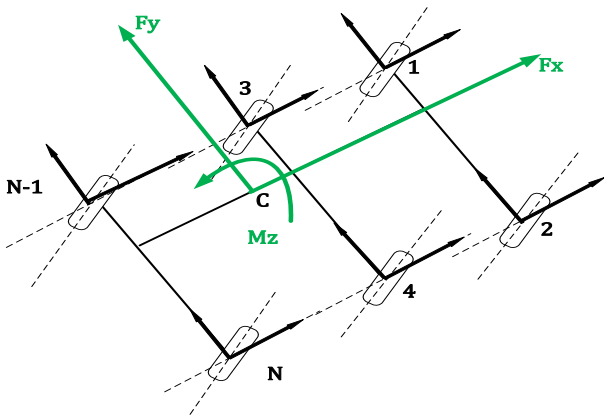


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7 / 12

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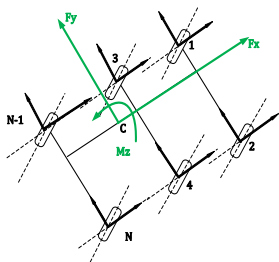


# Force Distributor

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## Objective

Distribute desired  $F_x$ ,  $F_y$  and  $M_z$  to each wheel tire module.



## Optimization Problem:

$$\arg \min_f J = f^T W f$$

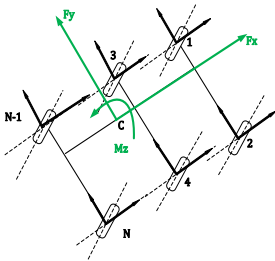
$f$  is a vector containing all  $f_{x,i}$  and  $f_{y,i}$ ;  $i = 1, ..N$ .

# Force Distributor

7 / 12

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## Location of each wheel:

$$M f = F_d$$

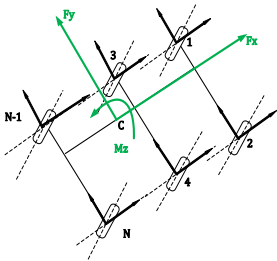
$F^d$  is the desired control signal from the outer body control.

# Force Distributor

7 / 12

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## Location of each wheel:

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$F^d$  is the desired control signal from the outer body control.

## Limitation on vertical load:

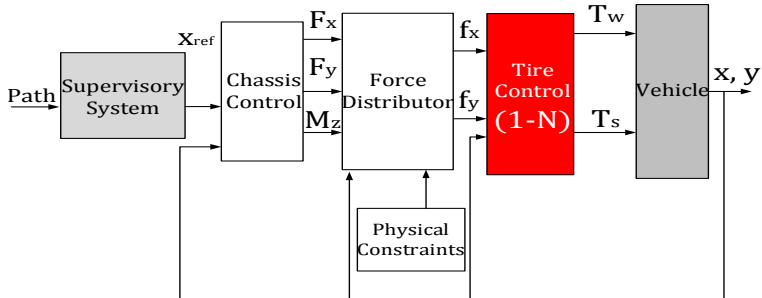
$$G f \leq h$$

# Tire Control

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## Objective

Determine the steering and driving actuation for generating the desired tire forces.

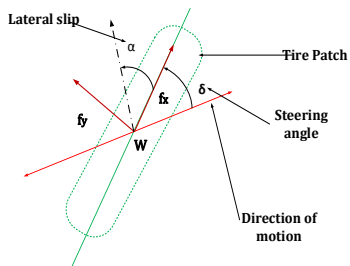


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8 / 12

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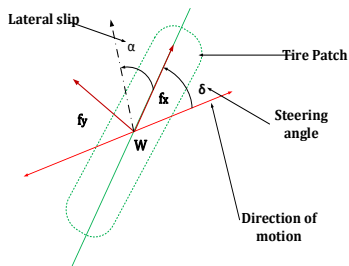


# Tire Control

8 / 12

## Objective

Determine the steering and driving actuation for generating the desired tire forces.



## Wheel & Tire Dynamics:

- Nonlinear slip dynamics and steer-by-wire dynamics:

$$\dot{x}_w = f_w(x_w) + g_w u_w, \quad y_w = h(x_w)$$

- Inputs( $u_w$ ): Steering torque and wheel torque
- Output( $y_w$ ): wheel forces in longitudinal and lateral direction.

# Tire Control

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Diffeomorphic Transformation:

$$\xi = \Phi(x_w), \quad \dot{\xi} = b(\xi) + A(\xi)u_w$$

State Feedback Structure:

$$u_w = A^{-1}(\xi)[v - b(\xi)]$$

Virtual Control input:

$$\dot{\xi} = I \xi + b v_w$$

Design  $v_w$  with linear control technique.

**Closed loop nonlinear system is exponentially stable.**



# Simulation Results

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## Simulation Setting

Six wheeled vehicle with independent actuation on each wheel.

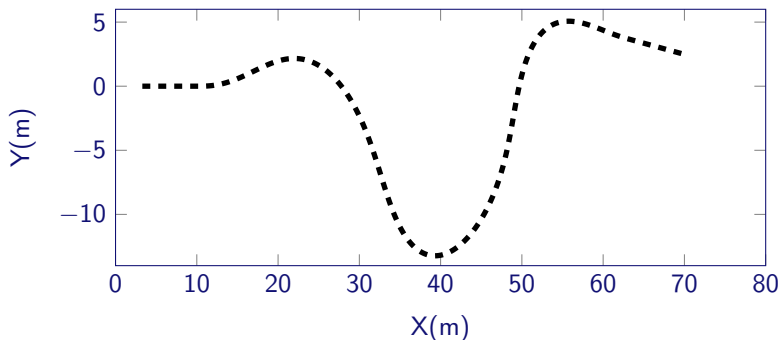


Figure: Reference Route

# Simulation Results

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Six wheeled vehicle with independent actuation on each wheel.

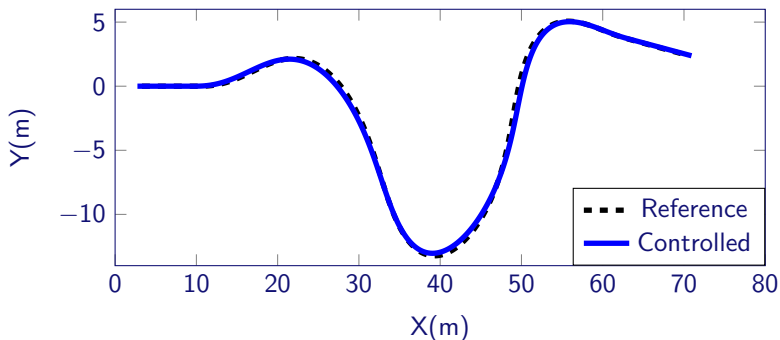


Figure: Closed loop tracking

# Simulation Results

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## Simulation Setting

Six wheeled vehicle with independent actuation on each wheel.

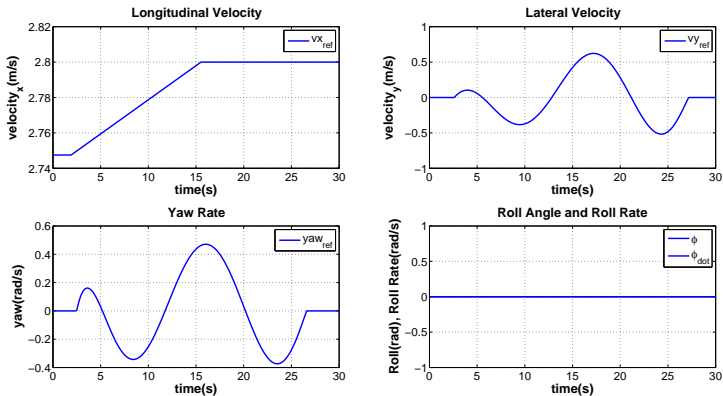


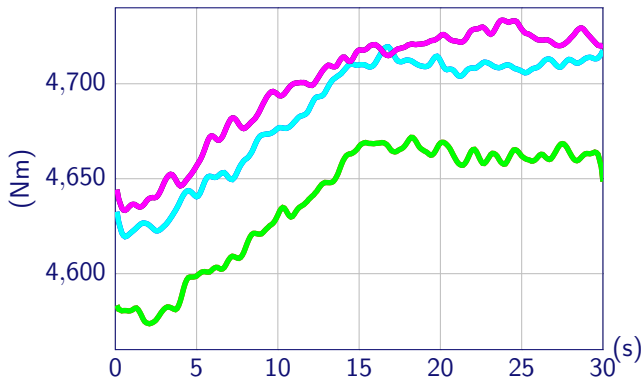
Figure: Reference state trajectory

# Simulation Results

9 / 12

## Simulation Setting

Six wheeled vehicle with independent actuation on each wheel.



**Figure:** Wheel torques for the wheels on each of three axles. **Green:**=Front Axle; **Cyan:**=Center Axle; **Purple:**=Rear Axle.

# Simulation Results

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## Simulation Setting

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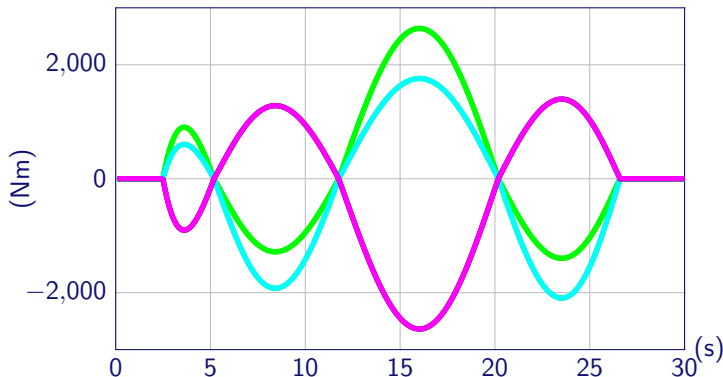


Figure: Steer torques for the wheels on each of three axles. Green:=Front Axle; Cyan:=Center Axle; Purple:=Rear Axle.

# Conclusions

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- A three-stage cascade control scheme which separates the dynamics of chassis from each wheel and tire.
- The design is generic in the sense of incorporating multiple wheel & tire modules.
- Incorporating steering torque as control variable allows for handling large steering angle.

# Future Recommendations

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- Observer based control design in case of limited sensor measurements.
- Addressing robustness issue regarding model-plant mismatch, other uncertainties.
- Including actuator limits.

# Thank You

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QUESTIONS ?