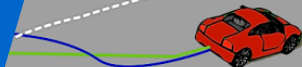


# Optimal Trajectory Tracking Control for Automated Guided Vehicles

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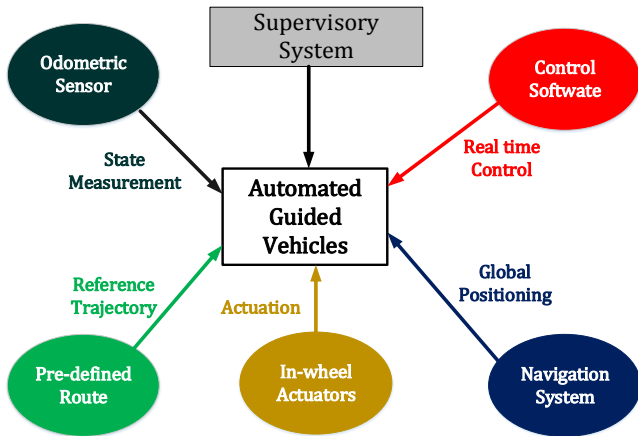
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# Automated Guided Vehicles (AGVs)

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Supervised autonomous driving in pre-defined route.



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Di erent application, identical driving principal.

# Research Question

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How to develop a control strategy for automated guided vehicles which tracks a pre-de ned trajectory ?

Features:

1. Generic for any kind of AGV with arbitrary number of wheels.
2. Handling severe cornering maneuver.
3. Carrying heavy load in elevated or banked road surface.



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# System Overview

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Vehicle as multibody system.

## Observation

Separate the control problem of each wheel & tire module from chassis.

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## Observation

Separate the control problem of each wheel & tire module from chassis.

# Controller Architecture

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Cascade Control structure.

# Controller Architecture

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Determine the optimal longitudinal, lateral body force and also the yaw moment to be applied to the center of mass of the chassis.

# Controller Architecture

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Distribute the desired forces and moment from the chassis controller over  $N$  controllable wheels, under physical constraints.

# Controller Architecture

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Determine the control input for each in-wheel actuator to track desired wheel-forces.

# Chassis Control

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## Objective

Determine the desired  $\mathbf{d}_b := [F_x \ F_y \ M_z]^T$  for given  $x_{\text{ref}}(t)$ .



# Chassis Control

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## Objective

Determine the desired  $\mathbf{u}_b := [F_x \ F_y \ M_z]^T$  for given  $\mathbf{x}_{\text{ref}}(t)$ .

### Considerations:

- | Chassis as rigid body.
- | Including load, new center of mass is calculated.
- | Nonlinear dynamics of the chassis  
 $\dot{\mathbf{x}}_b = \mathbf{f}_b(\mathbf{x}_b; \mathbf{u}_b)$ .
- |  $\mathbf{x}_b$  includes longitudinal velocity , lateral velocity , yaw rate, roll, roll rate .

# Chassis Control

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## Objective

Determine the desired  $\mathbf{u}_b := [F_x \ F_y \ M_z]^T$  for given  $\mathbf{x}_{\text{ref}}(t)$ .

### Steps:

- | Divide  $\mathbf{x}_{\text{ref}}(t)$  into finite segments.
- | Linearize the model for each segment.
- | Apply receding horizon LQ optimal control.

# Chassis Control

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## Objective

Determine the desired  $F_x$ ,  $F_y$  and  $M_z$  for  $x_{\text{ref}}(t)$  with  $t \in [t_k; t_{k+1}]$ .

### Steps:

- Divide  $x_{\text{ref}}(t)$  into finite segments.
- Linearize the model for each segment.
- Apply receding horizon LQ optimal control.

### Cost Functional for $k^{\text{th}}$ segment:

$$J(x_b; x_{\text{ref}}; u_b) = e^T(t_{k+1}) Q_f e(t_{k+1}) + \int_{t_k}^{t_{k+1}} [e^T(t) Q e(t) + u_b^T(t) R u_b(t)] dt$$

$$\text{Tracking error}(t) := x_{\text{ref}}(t) - x_b(t)$$

# Chassis Control

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ARE based State Feedback:

$$A^T K + K A - K B R^{-1} B^T K + Q = 0;$$

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$$r_b(t) = [A^T - K B R^{-1} B^T] r_b(t) + Q x_{ref}(t);$$

$$r_b(t_{k+1}) = Q_f x_{ref}(t_{k+1})$$

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Control Input:

$$u_{b;opt}(t) = -R^{-1} B^T [K x_b(t) + r_b(t)];$$

# Force Distributor

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## Objective

Distribute desired  $F_x$ ,  $F_y$  and  $M_z$  to each wheel tire module.

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Optimization Problem:

$$\arg \min_f J = f^T W f$$

$f$  is a vector containing all  $f_{x;i}$  and  $f_{y;i}$ ;  $i = 1 ; ::N$ .

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Limitation on vertical load:

$$G f \leq h$$

# Tire Control

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Determine the steering and driving actuation for generating the desired tire forces.

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### Wheel & Tire Dynamics:

- | Nonlinear slip dynamics and steer-by-wire dynamics:

$$\underline{x}_w = f_w(x_w) + g_w u_w; \quad y_w = h(x_w)$$

- | Inputs( $u_w$ ): Steering torque and wheel torque
- | Output( $y_w$ ): wheel forces in longitudinal and lateral direction.

# Tire Control

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Di eomorphic Transformation:

$$\dot{x}_w = A(x_w) x_w + b(x_w) u_w$$

State Feedback Structure:

$$u_w = -A^{-1}(x_w) [v + b(x_w)]$$

Virtual Control input:

$$\dot{v} = -I + b v_w$$

Design  $v_w$  with linear control technique.

Closed loop nonlinear system is exponentially stable.



# Simulation Results

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## Simulation Setting

Six wheeled vehicle with independent actuation on each wheel.

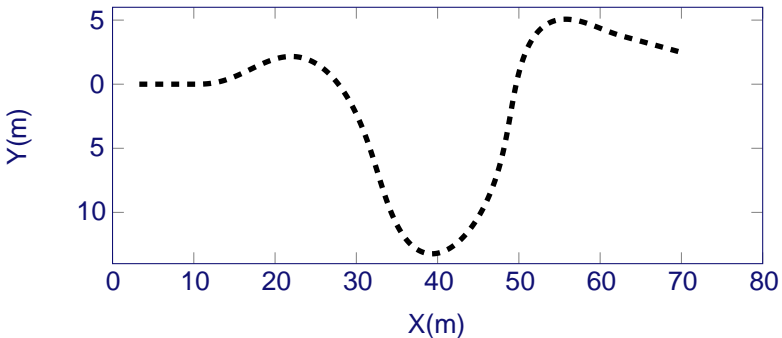


Figure: Reference Route

# Simulation Results

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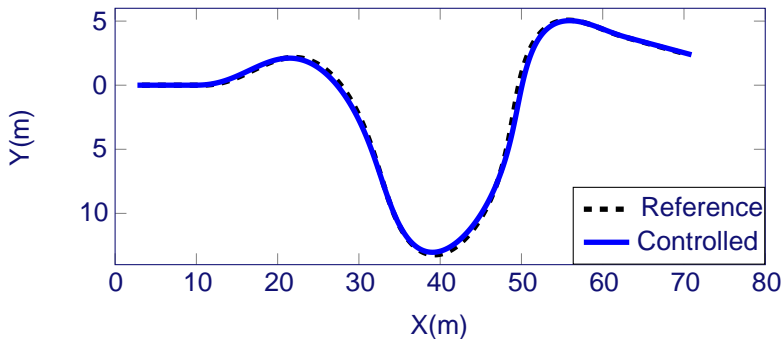


Figure: Closed loop tracking

# Simulation Results

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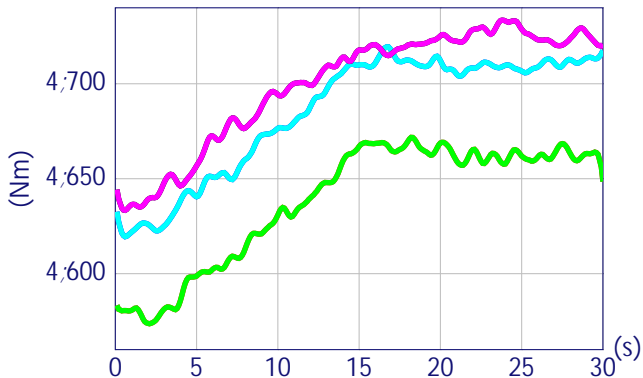
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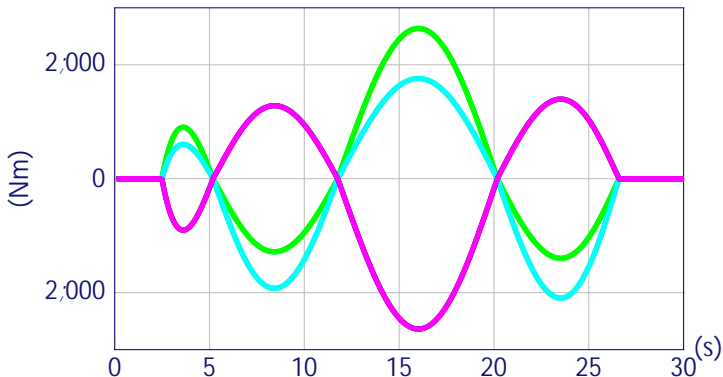
**Figure:** Wheel torques for the wheels on each of three axles. **Green:**:=Front Axle; **Cyan:**:=Center Axle; **Purple:**:=Rear Axle.

# Simulation Results

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**Figure:** Steer torques for the wheels on each of three axles. **Green:**:=Front Axle; **Cyan:**:=Center Axle; **Purple:**:=Rear Axle.

# Conclusions

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- A three-stage cascade control scheme which separates the dynamics of chassis from each wheel and tire.
- The design is generic in the sense of incorporating multiple wheel & tire modules.
- Incorporating steering torque as control variable allows for handling large steering angle.

# Future Recommendations

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- Observer based control design in case of limited sensor measurements.
- Addressing robustness issue regarding model-plant mismatch, other uncertainties.
- Including actuator limits.

# Thank You

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QUESTIONS ?