Supplementary Information for
"Are choices between risky options predictable?"

Angela Fontan[1*] Vladimir Cvetkovic[2] Pawel Herman[3] Marco Molinari[4]
Karl H. Johansson[1]

Contents

List of Supplementary Figures

List of Supplementary Tables

*Corresponding author.
[1]Div. Decision and Control Systems; [2]Div. Resources, Energy and Infrastructure; [3]Div. Computational Science and Technology; [4]Div. Applied Thermodynamics and Refrigeration. KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden, E-mail: {angfon,vdc,paherman,marcomo,kallej}@kth.se. The authors are also affiliated with Digital Futures.

Notation

Abbreviation	Meaning
PT	Prospect Theory (the same notation is used to refer to Cumulative Prospect Theory)
TPB	Theory of Planned Behaviour
RUM	Random Utility Model
cRUM	Contextualised Random Utility Model
pdf	Probability density function
cdf	Cumulative distribution function
log	Natural logarithm

Notation	Meaning
cRUM	
A, B	Prospects (or options)
V_A, V_B	Subjective (or raw) utilities of prospect A, B
$V_n = \lvert V_\text{A}\rvert + \lvert V_\text{B}\rvert$	L1 norm for the vector $[V_\text{A}, V_\text{B}] \in \mathbb{R}^2$
$\zeta_\text{A} = V_\text{A}/V_n$, $\zeta_\text{B} = V_\text{B}/V_n$	Contextualised utilities (bounded in $[-1, 1]$), representing attitude toward prospect A, B
β	Choice parameter
$\beta(\zeta_\text{A} - \zeta_\text{B})$	Intention of choosing A over B
P_A (resp., P_B)	Probability of choosing prospect A over prospect B (resp., B over A)
$\beta \sim LN[\mu, \sigma]$	Log-normal distribution for β, where μ and $\sigma > 0$ are the mean and standard deviation of $\log(\beta) \sim \mathcal{N}[\mu, \sigma]$ (where $\mathcal{N}$ represents the normal distribution)
Statistical indicators	
$r \in [-1, 1]$	Pearson's correlation coefficient
MSE ≥ 0	Mean squared error
p-value ≥ 0	p-value for the t-test, i.e., for the null hypothesis that the difference between modelled and observed P_A data comes from a normal distribution with mean equal to zero and unknown variance

1 Problem Formulation

Predicting human choices is a challenging problem with implications well beyond behavioural economics, ranging from economics to politics, from transportation to lifestyle choices, and in its most general formulation the problem can be expressed in the form of the question *How do people make choices among different alternatives?*

A standard hypothesis in the literature, consistent with, e.g., expected utility theory [2, 3] or random utility theory [4], is that a decision-maker (in this manuscript we will use the terms “decision-maker” and “subject” interchangeably) chooses the alternative among two prospects that maximises his/her utility, usually expressed in monetary units. It is then natural to assume that this decision-making process considers two types of variables:

- *Exogenous* variables, which are observable, for instance attributes on the alternatives which can be expressed in terms of outcomes and associated probabilities;
- *Endogenous* variables, which are not observable and vary among decision-makers.

A framework used to model discrete choices under assumption of utility-maximising behaviour is given by the Random Utility Model (RUM), described in the next section.

1.1 Random Utility Model (RUM)

The RUM is a cornerstone of behavioural economics for decisions on alternatives with uncertain outcomes, whose aim is to quantify choices by decision-makers among discrete alternatives [4, 5, 6, 7, 8]. The key assumption of RUM is that decision-maker’s preferences can be described by a utility function where she/he chooses the alternative with highest perceived value, i.e., highest utility. The utility depends on exogenous (observable) attributes and endogenous (unobservable, subjective) attributes which randomise the actual choice, hence RUM computes the probability of choosing a given alternative.

Assume that a decision-maker needs to make a choice between N alternatives, where in this work we consider $N = 2$ and denote the alternatives by A and B. The utility (or subjective valuation) assigned to each alternative is denoted by V_{A} and V_{B}, respectively, and depends both on observable and unobservable attributes, denoted by X and X^*, respectively, so that $V_{\mathrm{A}} = f(X, X^*)$. By representing X^* as a random variable, while we are not able to model choices with certainty, we can compute a probability of the decision-maker choosing a given alternative. The way we model this probability is through a Sigmoid or Logit function (Def. 1).

Definition 1 (RUM)**.** *Let* V_{A} *and* V_{B} *be the utilities associated with the discrete alternatives* A *and* B*, respectively. Then, the probability of choice (A or B) is given by:*

$$P_{\mathrm{A}} = (1 + \exp(-\beta(V_{\mathrm{A}} - V_{\mathrm{B}})))^{-1}\,, \quad P_{\mathrm{B}} = (1 + \exp(-\beta(V_{\mathrm{B}} - V_{\mathrm{A}})))^{-1} = 1 - P_{\mathrm{A}}, \tag{1}$$

where β *is a weight or calibration parameter.*

1.2 The weight parameter in RUM

The weight parameter β is typically calibrated for the set of data at hand. Briefly, one can for instance consider the following cases:

1. Deterministic case: A constant β is associated to a decision-maker. Given a dataset of observed binary choices made by a decision-maker for specific outcomes (A, B), β can be estimated using Maximum Likelihood Estimation, by modelling the response variable using a logistic regression model.
2. Stochastic case: A mixing distribution of β is associated with a population of decision-makers. Given a dataset of observed binary choices made by each decision-maker in a population for specific outcomes (A, B), a mixing distribution of β, specifying the distribution of the weight parameter over people, can be estimated using a mixed logit model [9].

We argue that the current treatment of β in the literature lacks coherency, which limits the predictive power of RUM. In particular, there does not seem to be consensus in the notation for the parameter β; it is referred to in the literature in different ways, such as a "sensitivity parameter" [10], "steepness of S-shaped function" [11], "free sensitivity parameter" [12], "precision parameter" [13], "inverse rationality parameter" [14], "strength of preference parameter" [15]. Furthermore, this diverse nomenclature indicates that β is understood as an endogenous parameter, even if in the common RUM formulation β is explicitly dependent on exogenous variables, i.e., the magnitude and units of the utilities $V_{\mathrm{A}}, V_{\mathrm{B}}$.

To summarise: The range of values for β cannot be transferred from one experiment to another (i.e., there is no obvious value of β to use for predicting choices across experimental settings), and the expression for (binary) choice probability P_{A} is limited by the context-dependent parameter β. Hence, we argue that separation of exogenous and endogenous variables is imperative, not only for improving predictability of economic choices, but also for their interpretability. To resolve this problem we propose a new method for computing the choice probability, called contextualised RUM (cRUM).

1.3 Contributions and outline

Our main contributions can be briefly stated as follows:

1. We propose cRUM, a model for choice probability with endogenous choice parameter β;
2. We validate cRUM and demonstrate the endogenous property of β in two consecutive steps, each one using independent data on discrete choice experiments.

The Supplementary Information is organised as follows. First, we introduce cRUM (Supplementary Section 2.1) and discuss how to derive a stochastic model for β to capture variability of choices in a population of decision-makers (Supplementary Section 2.2). Afterwards, we present how to compute subjective utilities in cRUM (Supplementary Section 2.3) and introduce the notion of framing effect (Supplementary Section 2.4). Finally, we introduce the datasets on discrete choice experiments used to test cRUM (Supplementary Sections 3.1 and 3.2): we use experimental studies on the framing effect to test the inferred population-distribution for the parameter β (Supplementary Section 3.3), and we test its population-representative value across several datasets containing prospect choice experiments with different experimental settings (Supplementary Section 3.4).

2 Contextualisation by means of normalisation: From RUM to cRUM

2.1 Contextualised Random Utility Model (cRUM)

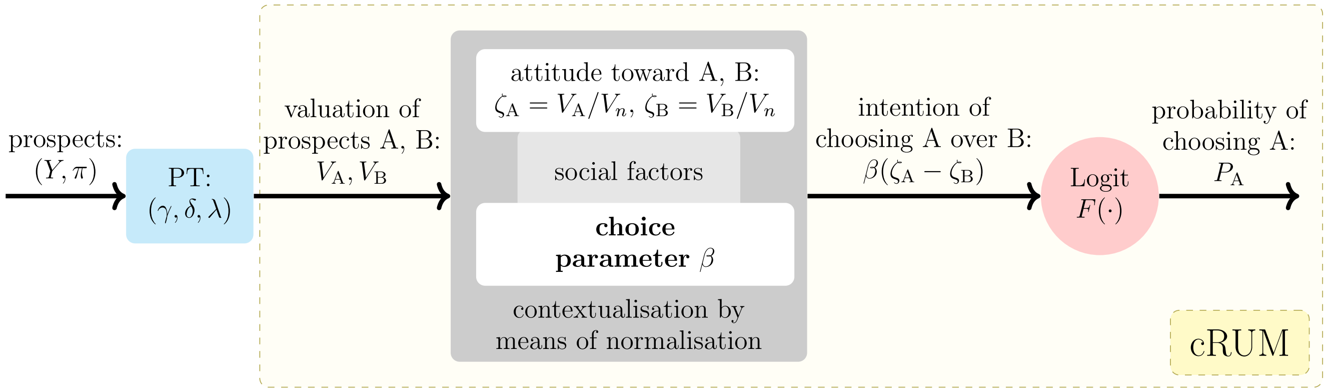
In spite of its wide use, the common expression in eq. (1) of the Logistic model for binary choices P_{A} is fundamentally limited by the context-dependent parameter β. There is no obvious value of β from the literature that can be utilised for predicting choices *across experimental settings*, where nominal values significantly vary. This section provides a computational viewpoint of cRUM, a new method for computing the choice probability. The definition is proposed in eqs. (1)-(2) in the main text, and is reported in what follows for completeness. Supplementary Fig. 1 provides an illustration.

Definition 2 (cRUM). *Let V_{A} and V_{B} be the utilities associated with the discrete alternatives* A *and* B*, respectively. The* attitudes *of a decision-maker toward prospect* A *and prospect* B *are given by the normalised utilities*

$$\zeta_{\mathrm{A}} = \frac{V_{\mathrm{A}}}{|V_{\mathrm{A}}| + |V_{\mathrm{B}}|} \quad \text{and} \quad \zeta_{\mathrm{B}} = \frac{V_{\mathrm{B}}}{|V_{\mathrm{B}}| + |V_{\mathrm{B}}|},$$

respectively. The intention *of a decision-maker to choosing prospect* A *over prospect* B *is given by*

$$\beta(\zeta_{\mathrm{A}} - \zeta_{\mathrm{B}}) = \beta \frac{V_{\mathrm{A}} - V_{\mathrm{B}}}{|V_{\mathrm{A}}| + |V_{\mathrm{B}}|}, \tag{2}$$



Supplementary Fig. 1: Illustration of cRUM, a framework for predictive modelling of choice between prospects A and B (Def. 2). Contextualisation of valuations is ensured by normalisation using the L1 norm $V_n = |V_A| + |V_B|$.

where β *is a dimensionless scalar referred to as the* choice parameter. *The* contextualised probability *by a decision-maker of choosing prospect* A *over prospect* B *is given by*

$$P_A = (1 + \exp((-\beta(\zeta_A - \zeta_B)))^{-1} = \left(1 + \exp\left(-\beta \frac{V_A - V_B}{|V_A| + |V_B|}\right)\right)^{-1}, \tag{3}$$

and of choosing prospect B *over prospect* A *is given by* $P_B = 1 - P_A$.

Eq. (3) proposes to incorporate contextualisation in valuation of prospects into a Logit type choice function. The novelty of eq. (3) lies in the normalisation (implemented by means of L1 norm), as a method to draw comparisons between the intentions towards the two alternative prospects, A and B.

Inverting eq. (3), one can obtain the following expression for β as a function of observed choice probability P_A and contextualised utilities ζ_A, ζ_B[1]:

$$\beta = \begin{cases} 0, & \text{if } P_A = \frac{1}{2} \text{ (or, equivalently, } \zeta_A - \zeta_B = 0) \\ \frac{1}{\zeta_A - \zeta_B} \log\left(\frac{P_A}{1 - P_A}\right) \geq 0, & \text{otherwise.} \end{cases} \tag{4}$$

In this new context, and in line with TPB theory [16], we name β as the choice (or control) parameter, which can be interpreted as quantifying control, i.e., how sure a decision-maker is when making a choice: $\beta \geq 0$, where $\beta \to 0$ implies $P_A = \frac{1}{2}$ and $\beta \to +\infty$ implies $P_A = 1$.

2.2 The choice parameter in cRUM

2.2.1 Connection to perceived probability of unlikely events

According to Def. 2, the choice parameter β does not explicitly depend on the prospect utilities as in the classical formulation, and thus can be considered *endogenous*. Intuitively, this implies that suitable psychological and/or neurological evidence should be sufficient to infer a distribution for β. To this end, we propose to interpret the choice parameter in terms of lowest perceived probability of unlikely events (i.e., perception that an event, e.g., choosing A, is unlikely); this intuition comes directly from Def. 2, as we explain in the following.

From eq. (2) it follows that the intention of choosing prospect A over prospect B is bounded in the interval $[-\beta, \beta]$. Indeed, a sure choice of prospect A, defined as choice associated with sure probabilities $P_A = 1, P_B = 0$, is represented by $\zeta_A = 1, \zeta_B = 0$, yielding $\beta(\zeta_A - \zeta_B) = \beta$; similarly, a sure choice of prospect B, defined as choice associated with sure probabilities $P_A = 0, P_B = 1$, is represented by $\zeta_A = 0, \zeta_B = 1$, yielding $\beta(\zeta_A - \zeta_B) = -\beta$. Theoretically, using eq. (3) the interval $[-\beta, \beta]$ for the intention is associated with the interval $[0, 1]$ for the probability of choosing prospect A over B. However, in practice,

[1]Note that when $P_A = \frac{1}{2}$, we assume $\zeta_A - \zeta_B = 0$, and we define $\beta = 0$. This case represents a situation where the choice between prospects is ambiguous, which we treat as a purely random choice: the utility of prospect A (ζ_A) is equal to the utility of prospect B (ζ_B).

the definition of zero probability depends on what humans *perceive* as near-zero probability, or probability of unlikely event.

2.2.2 Derivation of a stochastic model for the choice parameter in cRUM

Variability in a population can be captured by assuming a stochastic model for β with pdf defined on $(0, +\infty)$. In this work we assume that β follows a log-normal distribution, i.e., $\beta \sim LN[\mu, \sigma]$ where μ and $\sigma > 0$, or, equivalently, $\beta = \exp(\mu + \sigma Z)$ where $Z \sim \mathcal{N}[0, 1]$. We use the average of the distribution, denoted by $E[\beta]$, to serve as a population-representative value for the choice parameter β, fixed across decision-makers.

According to the interpretation provided in Supplementary Section 2.2.1, we use human-social probability perception evidence available in the literature to infer a log-normal distribution for β. We consider two constraints: first, we refer to *perceptual numerosity experiments* to estimate $E[\beta]$; then, we refer to *human perception of societal risks* to identify a plausible range for β that accounts for a sufficiently high level of certainty.

Derivation of a population-representative value Psychological studies of probability perception [17, 18] suggest 1/1 000 as perceptual frequency resolution for a group of decision-makers, from which we derive an average (or population-representative) value for the lowest detectable probability. Using this criterion in eq. (4), with $\zeta_{\mathrm{A}} - \zeta_{\mathrm{B}} = -1$ and $P_{\mathrm{A}} = 1/1\,000$, yields the following definition.

Definition 3. *The* population-representative *value of the choice parameter β corresponding to lowest* perceived probability of unlikely event *(defined as* perception of near-zero probability*) of* 1/1 000 *is given by:*

$$E[\beta] = \log\left(\frac{1 - 0.001}{0.001}\right) = \log(999) \approx 7, \tag{5}$$

Observe that from eq. (3), $E[\beta]$ is equal to the log of odds of prospect B over A, that is, as the chances of B (likely event) happening divided by the chances of A (likely event) happening: $E[\beta] = \log\left(\mathrm{odds}_{\mathrm{B\,vs\,A}}\right) = \log\left(1/\mathrm{odds}_{\mathrm{A\,vs\,B}}\right)$.

Derivation of a log-normal distribution Studies on human perception of societal risks (see main manuscript), identify 1/1 000 000 as a generally accepted frequency threshold for (in)tolerable risk, both from the individual and societal risk perspectives.

The constraints we impose to derive the estimates of μ and σ are then given as:

- On average the population has lowest perceived probability of unlikely events equal to 1/1 000 (eq. (5));
- 95% of the population has lowest perceived probability of unlikely events $\geq 1/1\,000\,000 = 10^{-6}$.

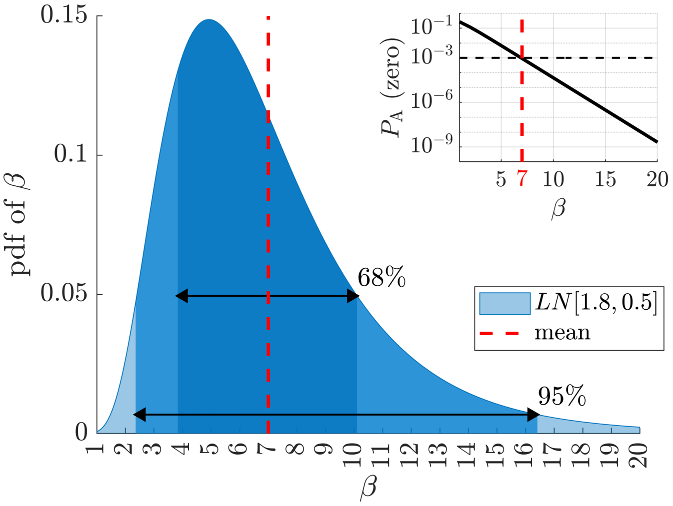
Given that $\zeta_{\mathrm{A}} - \zeta_{\mathrm{B}} = -1$ and, consequently, that the lowest perceived probability of unlikely events (considered as the perception of near-zero probability) is given by $P_{\mathrm{A}} = (1 + \mathrm{e}^{\beta})^{-1}$, the second constraint implies that

$$\begin{aligned} 0.95 = P\left[P_{\mathrm{A}} \geq 10^{-6}\right] &= P\left[(1 + \mathrm{e}^{\beta})^{-1} \geq 10^{-6}\right] = P\left[\frac{1 - 10^{-6}}{10^{-6}} \geq \mathrm{e}^{\beta}\right] \\ &= P\left[\beta \leq \log\left(\frac{1 - 10^{-6}}{10^{-6}}\right)\right] = P\left[\beta \leq \beta_{95\%}\right] \text{ with } \beta_{95\%} = \log\left(\frac{1 - 10^{-6}}{10^{-6}}\right). \end{aligned}$$

All in all, the two constraints imposed in the stochastic model can be formally rewritten as

$$\begin{cases} E[\beta] = \exp\left(\mu + \frac{\sigma^2}{2}\right) := 7, & \text{where } E[\beta] = 7 \text{ is deduced in eq. (5)} \\ P[\beta \leq \beta_{95\%}] = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{\log(\beta_{95\%}) - \mu}{\sigma\sqrt{2}}\right)\right) := 0.95, & \text{where } \beta_{95\%} = \log\left(\frac{1 - 10^{-6}}{10^{-6}}\right) \end{cases} \tag{6}$$

Solving the system of equations (6) yields $\mu = 1.8$, $\sigma = 0.5$, and consequently we define the log-normal distribution for β as follows.



Supplementary Fig. 2: Log-normal distribution of β, $\beta \sim LN[1.8, 0.5]$ (eq. (7)). Legend: The 68% and 95% confidence intervals are highlighted with darker blue areas. A dashed red line indicates the arithmetic mean of the distribution $E[\beta] = 7$ (eq. (5)), corresponding to the population-representative value for the choice parameter β.

Definition 4. *The* population-distribution *of the choice parameter β corresponding to lowest perceived probability of unlikely events of* 1/1 000 *on average, with perception of 95% of the population being* $\geq$ 1 000 000*, is given by:*

$$\beta \sim LN[1.8, 0.5]. \tag{7}$$

Supplementary Figure 2 illustrates the distribution and highlights the 68% and 95% confidence intervals.

2.3 Computation of valuation by means of Prospect Theory

In Def. 2 it is assumed that valuations of prospects are known for choice problems; therefore, a necessary step is to first compute the valuations of prospects from the collected data. In this section we briefly explain how to compute utilities using Prospect Theory (PT)[2] in a setting where a decision-maker needs to make a choice between two alternative prospects A and B (the reader is referred to [19] for details). Let prospects A and B be written as:

$$\mathrm{A}: \{(Y_{\mathrm{A}_1}, \pi_{\mathrm{A}}), (Y_{\mathrm{A}_2}, 1-\pi_{\mathrm{A}})\} \qquad \mathrm{B}: \{(Y_{\mathrm{B}_1}, \pi_{\mathrm{B}}), (Y_{\mathrm{B}_2}, 1-\pi_{\mathrm{B}})\}$$

where Y_{A_1} and Y_{A_2} are outcomes for prospect A with probabilities π_{A} and $1-\pi_{\mathrm{A}}$, respectively, and similarly for prospect B. A decision-maker chooses a prospect based on the perceived values of alternatives A and B, namely, V_{A} and V_{B}. According to PT, the utilities V_{A} and V_{B} are computed by:

$$V = \begin{cases} U(Y_1)\, w(\pi) + U(Y_2)\,(1-w(\pi)) & \text{for gain or loss prospects} \\ U(Y_1)\, w(\pi) + U(Y_2)\, w(1-\pi) & \text{for mixed prospects} \end{cases} \tag{8a}$$

$$U(Y) = \begin{cases} (Y-Y_0)^{\delta+} & \text{if } Y \geq Y_0 \\ -\lambda(Y_0-Y)^{\delta-} & \text{if } Y < Y_0 \end{cases} \tag{8b}$$

$$w(\pi) = \frac{\pi^{\gamma}}{\left(\pi^{\gamma} + (1-\pi)^{\gamma}\right)^{1/\gamma}} \tag{8c}$$

where Y_0 is a reference value and V, Y_1, Y_2, and π pertain to A or B. Gain (or, positive) prospects are defined as $Y_1 > Y_2 \geq Y_0 \geq 0$, loss (or, negative) prospects as $Y_1 < Y_2 \leq Y_0 \leq 0$, while mixed prospects as $Y_1 < Y_0 < Y_2$. The function $U(Y)$ is called the utility function, while $w(\pi)$ the weighting function: we choose the functionals proposed in the classical work by Kahneman and Tversky [19], but there are several studies addressing the specific functional forms of $U(x)$ of eq. (8b) and $w(\pi)$ of eq. (8c), see, e.g.,

[2] In this work we use PT to refer to Cumulative Prospect Theory [19].

[20]. Note also that the general formulation of $U(Y)$ includes two different parameters for gain and loss prospects, i.e., δ^+ and δ^-, respectively. If not explicitly written, in this manuscript it is assumed that $\delta^+ = \delta^- = \delta$.

Selection of the positive PT parameters $(\lambda, \delta, \gamma)$ quantifies different risk and valuation biases. Specifically, $(\lambda, \delta, \gamma) = (1, 1, 1)$ implies a simple expected value without any biases, $(\lambda, \delta, \gamma) = (> 1, < 1, 1)$ implies an expected utility model without accounting for the risk bias, whereas $(\lambda, \delta, \gamma) = (> 1, < 1, < 1)$ includes both value and risk biases. In the literature on PT, the parameters in eqs. (8b)-(8c) are inferred from experiments; in a number of studies, only δ and γ are considered, with $\lambda = 1$ [20, 21, 15]. In this work, we denote by *standard PT parameters* the values $(\gamma, \delta, \lambda) = (0.65, 0.88, 2.25)$ (note: $\delta^+ = \delta^- = \delta$), from the calibration proposed in the classical work by Kahneman and Tversky [19][3].

2.4 The Framing Effect

The framing effect describes the difference in behaviour caused by "variations in the framing of acts, contingencies, and outcomes" [22], and it is now subject of many experimental studies, e.g., [23, 24, 25, 1, 26, 27] and references therein. The definition we adopt in this work is the following.

Definition 5 (Framing effect)**.** *Consider an experimental setting where decision-makers need to choose between a risky and a sure prospect, denoted by prospect A and prospect B, respectively. The framing effect of each decision-maker i is defined as the difference in the percentage of trials in which the risky prospect was chosen by i between the loss frame and the gain frame. It is denoted by: framing effect(i). The average framing effect across a population of decision-makers is defined as the mean:* $\frac{1}{n}\sum_{i=1}^{n}$ *framing effect(i), where n is the total no. of decision-makers.*

In the following we formulate the valuation of prospects illustrated in Supplementary Section 2.3 in the gain and loss frames, to ease the computation of the framing effect. We define the prospects in a gain and loss frame as follows, where $Y > 0$ and $\pi \in [0, 1]$:

$$\begin{cases} \text{gain frame} & \mathrm{A}: \{(Y, \pi), (0, 1-\pi)\} \qquad \mathrm{B}: \{(Y\pi, 1), (0, 0)\}, \\ \text{loss frame} & \mathrm{A}: \{(-Y, 1-\pi), (0, 1-\pi)\} \quad \mathrm{B}: \{(-Y(1-\pi), 1), (0, 0)\}. \end{cases} \tag{9}$$

Note that the sure prospect is created to match the expected value of the gamble, depending on framing. Using PT to compute the valuations V_{A} and V_{B} for prospects A and B in both frames (eq. (8) with $\delta^- = \delta^+ = \delta$), calculations held:

$$V_{\mathrm{A_G}} = Y^\delta w(\pi), \quad V_{\mathrm{B_G}} = Y^\delta \pi^\delta, \quad |V_{\mathrm{A_G}}| + |V_{\mathrm{B_G}}| = Y^\delta \left(\pi^\delta + w(\pi)\right), \quad \text{and}$$
$$V_{\mathrm{A_L}} = -\lambda Y^\delta w(1-\pi), \quad V_{\mathrm{B_L}} = -\lambda Y^\delta (1-\pi)^\delta, \quad |V_{\mathrm{A_L}}| + |V_{\mathrm{B_L}}| = \lambda Y^\delta \left(w(1-\pi) + (1-\pi)^\delta\right),$$

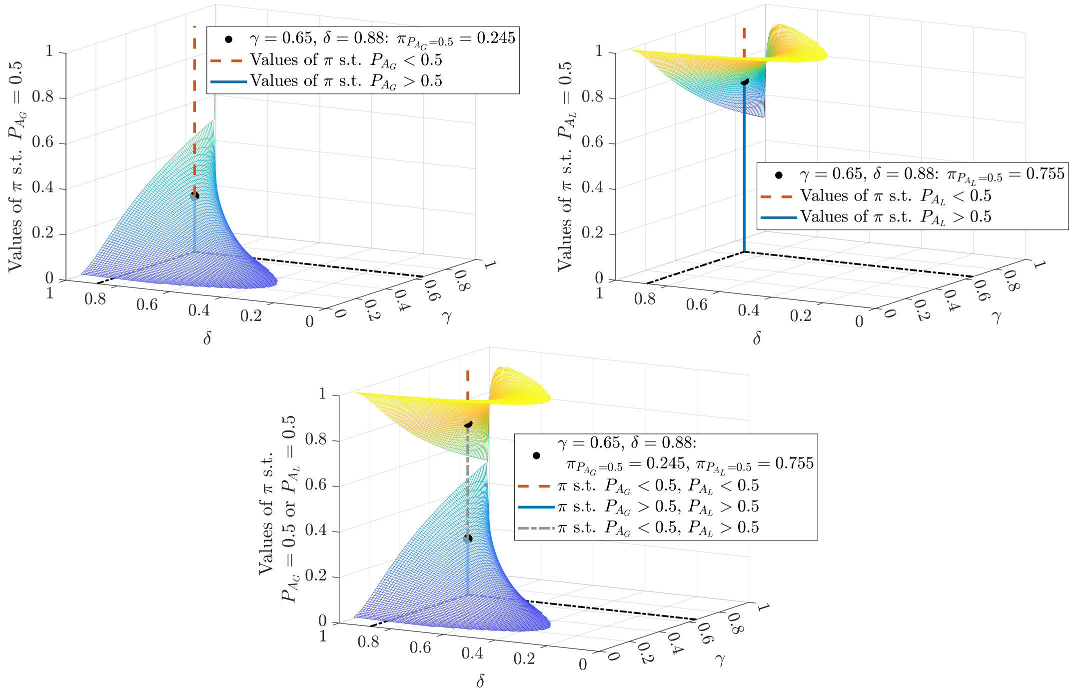
where $V_{\mathrm{A_G}}$ and $V_{\mathrm{B_G}}$ and $V_{\mathrm{A_L}}$ and $V_{\mathrm{B_L}}$ indicate the valuations for prospects A and B in the gain (G), and loss (L) frame. Using L1 normalisation, we obtain that the normalised utilities are given by:

$$\zeta_{\mathrm{A_G}} = \frac{w(\pi)}{w(\pi) + \pi^\delta}, \qquad \zeta_{\mathrm{B_G}} = \frac{\pi^\delta}{w(\pi) + \pi^\delta}, \qquad \zeta_{\mathrm{A_G}} - \zeta_{\mathrm{B_G}} = \frac{w(\pi) - \pi^\delta}{w(\pi) + \pi^\delta}$$
$$\zeta_{\mathrm{A_L}} = -\frac{w(1-\pi)}{w(1-\pi) + (1-\pi)^\delta}, \quad \zeta_{\mathrm{B_L}} = -\frac{(1-\pi)^\delta}{w(1-\pi) + (1-\pi)^\delta}, \quad \zeta_{\mathrm{A_L}} - \zeta_{\mathrm{B_L}} = -\frac{w(1-\pi) - (1-\pi)^\delta}{w(1-\pi) + (1-\pi)^\delta}.$$

Therefore, the probabilities of choosing the risky prospect A in the gain ($P_{\mathrm{A_G}}$) and loss ($P_{\mathrm{A_L}}$) frames are given by:

$$P_{\mathrm{A_G}} = \frac{1}{1 + \exp\left(-\beta \frac{w(\pi) - \pi^\delta}{w(\pi) + \pi^\delta}\right)}, \quad P_{\mathrm{A_L}} = \frac{1}{1 + \exp\left(\beta \frac{w(1-\pi) - (1-\pi)^\delta}{w(1-\pi) + (1-\pi)^\delta}\right)}. \tag{10}$$

[3] The work [19] identifies two different values for γ, i.e., $\gamma = 0.61$ for loss prospects and $\gamma = 0.69$ for gain prospects. For simplicity and to keep a low no. of parameters, we consider $\gamma = 0.65$, i.e., the average value, as the standard PT parameter.



Supplementary Fig. 3: Values of π that satisfy the conditions $P_{\mathrm{A_G}} = 0.5$ (left panel), $P_{\mathrm{A_L}} = 0.5$ (right panel), and $P_{\mathrm{A_G}} = 0.5$ or $P_{\mathrm{A_L}} = 0.5$ (bottom panel) as a function of γ and δ. The values $\gamma = 0.65$ and $\delta = 0.88$ correspond to the standard PT parameters. In particular, the bottom panel highlights the values of π such that $P_{\mathrm{A_G}} < 0.5 < P_{\mathrm{A_L}}$ (grey dash-dotted line), illustrating a positive framing effect (Supplementary Section 2.4 and Remark 1 for details).

Supplementary Figure 3 illustrates the values of π s.t. $P_{\mathrm{A_G}} \gtreqless 0.5$ and/or $P_{\mathrm{A_L}} \gtreqless 0.5$, as a function of the PT parameters γ and δ; note that, given γ, δ, for all π s.t. $P_{\mathrm{A_L}} > 0.5 > P_{\mathrm{A_G}}$ the framing effect is positive. In particular, when $\gamma = 0.65$ and $\delta = 0.88$ (i.e., values associated with standard PT parameters), we obtain that:

$$P_{\mathrm{A_G}} \begin{cases} < 0.5, & \pi > 0.245 \\ = 0.5, & \pi = 0.245 \\ > 0.5, & \pi < 0.245 \end{cases}, \qquad P_{\mathrm{A_L}} \begin{cases} < 0.5, & \pi > 0.755 \\ = 0.5, & \pi = 0.755 \\ > 0.5, & \pi < 0.755, \end{cases}$$

This implies that, according to PT, $P_{\mathrm{A_L}} > 0.5 > P_{\mathrm{A_G}}$ for all values of $\pi \in (0.245, 0.755)$ and standard PT parameters, meaning that choosing the risky prospect A is more likely in the loss than in the gain frame. From the analysis above, we can draw the following observations.

Remark 1. *Let the prospects* A *and* B *be given as in eq.* (9), *and assume that PT (eq.* (8)*) is used to compute valuations* $V_{\mathrm{A}}, V_{\mathrm{B}}$ *with PT parameters* $(\gamma, \delta, \lambda)$. *The following statements hold:*

(i) The probabilities of choosing the risky prospect A in the gain ($P_{\mathrm{A_G}}$) and loss ($P_{\mathrm{A_L}}$) frames do not depend on the amount Y and are given by eq. (10)*;*

(ii) According to PT (with standard PT parameters) the framing effect is positive for all $\pi \in (0.245, 0.755)$, *i.e.,* $P_{\mathrm{A_L}} - P_{\mathrm{A_G}} > 0$ *for all* $\pi \in (0.245, 0.755)$.

3 Application: Experimental results on discrete choices datasets

3.1 Data description

To test cRUM and demonstrate the endogenous property of β we use datasets on discrete choice experiments (details in Table 1). We consider two types of datasets:

1. Discrete choice experiments evaluating the framing effect across decision-makers: see DS-FR1 and DS-FR2 of Table 1 in the main manuscript. The information collected includes:

- Two sets of problems/trials and, for each problem, the description of each alternative structured as economic prospects (i.e., outcomes associated with probabilities of winning/losing). The problems across the two sets are the same, but in the first set they are posed in a gain frame, while in the second set they are posed in a loss frame;
- The response of each decision-maker for each trial.

Results are presented in Supplementary Section 3.3.

2. Discrete choice experiments across a population of decision-makers: see DS1–DS12m of Table 1 in the main manuscript. The information collected includes:

- A set of problems/trials and, for each problem, the description of each alternative structured as economic prospects (i.e., outcomes associated with probabilities of winning/losing);
- The choice probability, defined as the percentage of trials in which each alternative was chosen). Observe that this is an aggregated information for each trial across decision-makers (the single responses for each trial and decision-maker are unknown).

Results are presented in Supplementary Section 3.4.

The software used for the numerical simulations is MATLAB®; for reproducibility, we specify the seed for the MATLAB®random number generator as `rng('default')`.

3.2 Data extraction from the datasets of Table 1

In this section we detail the data extraction from the datasets of Table 1 in the main manuscript, considering first in Section 3.2.1 the datasets DS-FR1 and DS-FR2, on discrete choice experiments evaluating the framing effect across decision-makers, and then in Section 3.2.2 the datasets DS1–DS12m, on discrete choice experiments across a population of decision-makers.

3.2.1 Discrete choice experiments evaluating the framing effect across decision-makers

DS-FR1, De Martino et al. [24] The Methods section of the main manuscript already gives a brief introduction on the data from De Martino et al. We report the information in this paragraph for completeness. The work of De Martino et al. presents experimental data that combine framing and heterogeneity effects, obtained by studying variation between 20 subjects. The prospects presented to the subjects during the experiments, namely, a risky prospect, A, and a sure prospect, B, are defined according to eq. (9), from the following (16 possible) combinations of outcomes Y and probabilities π:

$$\begin{cases} \text{Gain frame.} \quad \mathrm{A_G}: \{(Y_i, \pi_j), (0, 1-\pi_j)\} \quad \mathrm{B_G}: \{(Y_i \pi_j, 1), (0, 0)\} \\ \text{Loss frame.} \quad \mathrm{A_L}: \{(-Y_i, 1-\pi_j), (0, \pi_j)\} \quad \mathrm{B_L}: \{(Y_i(\pi_j - 1), 1), (0, 0)\} \end{cases}$$
$$\text{where} \quad Y_i = (£25, £50, £75, £100) \text{ and } \pi_j = (0.2,\ 0.4,\ 0.6,\ 0.8), \quad \text{for all } i, j = 1, 2, 3, 4.$$

Note that by computing utilities V_A and V_B according to PT (standard PT parameters, Supplementary Section 2.4), out of the 16 combinations, calculations show that there are only 4 distinct pairs for the normalised utilities $\zeta_\mathrm{A}, \zeta_\mathrm{B}$:

$$(\zeta_{\mathrm{A_G}} - \zeta_{\mathrm{B_G}}, \zeta_{\mathrm{A_L}} - \zeta_{\mathrm{B_L}}) = \left(\frac{w(\pi_j) - \pi_j^\delta}{w(\pi_j) + \pi_j^\delta}, - \frac{w(1-\pi_j) - (1-\pi_j)^\delta}{w(1-\pi_j) + (1-\pi_j)^\delta} \right) \quad j = 1, \dots, 4$$
$$\Downarrow \text{ with standard PT parameters } \delta = 0.88,\ \gamma = 0.65$$
$$= \{(0.0345,\ 0.1243),\ (-0.0775,\ 0.1237),\ (-0.1237,\ 0.0775),\ (-0.1243,\ -0.0345)\}.$$

As expected, PT yields $\zeta_{\mathrm{A_L}} - \zeta_{\mathrm{B_L}} > \zeta_{\mathrm{A_G}} - \zeta_{\mathrm{B_G}}$.

The framing effect, i.e., the percentage difference in choice probability P_A in the loss vs gain frames is reported in [24] for each of the 20 subjects; in ascending order it writes {0.061,0.073, 0.083, 0.085, 0.085, 0.104, 0.104, 0.104, 0.145, 0.167, 0.176, 0.197, 0.219, 0.228, 0.25,0.28, 0.312, 0.312, 0.375, 0.384}. The lowest value is interpreted by De Martino et al. as the most rational decision-maker and the highest value as the least rational decision-maker.

DS-FR2, Diederich et al. [1] In the second experiment described in [1] participants had to choose between a gamble, A, and a sure prospect, B, with the sure prospect presented in either a gain or a loss frame, similarly to the study by De Martino et al. [24]. Four initial amounts, flanked by plus/minus one point amounts, and four probabilities of winning the gamble were selected. The initial amounts and probabilities of winning the gamble were paired together to form 48 unique gambles, with L and G denoting loss and gain frames, respectively:

$$Y_i = (19\,€, 20\,€, 21\,€, 39\,€, 40\,€, 41\,€, 59\,€, 60\,€, 61\,€, 79\,€, 80\,€, 81\,€),$$
$$\pi_j = (0.3, 0.4, 0.6, 0.7), \quad i, j = 1, 2, 3, 4$$
$$\begin{cases} \mathrm{A_G} : \{(Y_i, \pi_j), (0, 1-\pi_j)\}, & \mathrm{B_G} : \{(Y_i\pi_j, 1), (0, 0)\} \quad \text{(gain frame)} \\ \mathrm{A_L} : \{(-Y_i, 1-\pi_j), (0, \pi_j)\}, & \mathrm{B_L} : \{(-Y_i(1-\pi_j), 1), (0, 0)\} \quad \text{(loss frame)} \end{cases}$$

In the experiment, the authors tested the influence of different experimental conditions, i.e., 2 frames (gain, loss), 2 time limits (1s, 3s), 3 needs (0, 2500, 3500), on the framing effect.

In this work, we group these experimental conditions in 7 case studies denoted DS-FR2.# (Supplementary Table 1). The number of participants considered in [1] is 54, but in our work we exclude data from one participant due to a high number of undefined responses, marked 9999 or nan in the cvs file: indeed, the percentage of undefined responses of participant with ID 38 is around 40%, compared to a percentage smaller than 0.08% for all other 53 participants. Thus, we consider 53 participants.

Note again that by computing utilities V_A and V_B according to PT (with standard PT parameters, Supplementary Section 2.4), out of the 48 combinations there are only 4 distinct pairs for the normalised utilities $\zeta_\mathrm{A}, \zeta_\mathrm{B}$:

$$(\zeta_{\mathrm{A_G}} - \zeta_{\mathrm{B_G}}, \zeta_{\mathrm{A_L}} - \zeta_{\mathrm{B_L}}) = \left(\frac{w(\pi_j) - \pi_j^\delta}{w(\pi_j) + \pi_j^\delta}, -\frac{w(1-\pi_j) - (1-\pi_j)^\delta}{w(1-\pi_j) + (1-\pi_j)^\delta} \right) \quad j = 1, \ldots, 4$$
$$\Downarrow \text{ with standard PT parameters } \delta = 0.88, \gamma = 0.65$$
$$= \{(0.0334,\ 0.1301),\ (-0.0775,\ 0.1237),\ (-0.1237,\ 0.0775),\ (-0.1301,\ -0.0334)\}$$

As expected, PT yields $\zeta_{\mathrm{A_L}} - \zeta_{\mathrm{B_L}} > \zeta_{\mathrm{A_G}} - \zeta_{\mathrm{B_G}}$.

According to the analysis presented in Supplementary Section 2.4, we expect $P_{\mathrm{A_L}} > 0.5 > P_{\mathrm{A_G}}$ for all π_j, $j = 1, 2, 3, 4$, since $\pi_j \in [0.3, 0.7] \subset (0.245, 0.755)$ for all $j = 1, 2, 3, 4$. That is, observation of a positive framing effect (for each subject) is consistent with PT (Remark 1). However, from the results of the experimental study conducted by Diederich et al. there are subjects who exhibits a nonpositive framing effect (last two rows of Supplementary Table 1 and Supplementary Fig. 4(a)(top panels)), which is not consistent with PT. We expect these case studies to be more complex to explain/predict by the approach we introduce in our work.

3.2.2 Discrete choice experiments across a population of decision-makers

The data collected in DS1–DS12m were retrieved either directly from the respective publications or from publicly available datasets posted online by the corresponding authors. For reproducibility reasons, we detail in what follows the retrieval of data from specific (and more complex) datasets, namely, DS5, DS9, and DS12–DS12m of Table 1.

DS5, Lopes and Oden [21] All input data used in our study that define prospects are contained in Figure 1 of [21], referred in their work as "standard stimuli". The figure illustrates six groups of outcomes, which in our computations are numbered as 1=riskless, 2=rectangular, 3=peaked, 4=bimodal, 5=short

	DS-FR2.1	DS-FR2.2	DS-FR2.3	DS-FR2.4	DS-FR2.5	DS-FR2.6	DS-FR2.7
Session	all	1	2	3	1	2	3
Time Limit	$1s$, $3s$	$1s$	$1s$	$1s$	$3s$	$3s$	$3s$
Need Levels	0, 2500, 3000	0	2500	3000	0	2500	3000
Frame (1 gain, 0 loss)	1/0	1/0	1/0	1/0	1/0	1/0	1/0
Avg % of trials A is chosen in gain frame	52.3%	47.1%	51.8%	51.6%	52.7%	53.9%	56.6%
Avg % of trials A is chosen in loss frame	70.9%	72.8%	70.8%	73.9%	71.5%	67.5%	69.2%
Avg Framing effect	18.6%	25.7%	19.0%	22.3%	18.8%	13.7%	12.6%
No. of subjects with framing effect< 0	3 (5.7%)	1 (1.9%)	4 (7.5%)	6 (11.3%)	5 (9.4%)	8 (15.1%)	10 (18.9%)
No. of subjects with framing effect= 0	0 (0.0%)	3 (5.7%)	1 (1.9%)	0 (0.0%)	2 (3.8%)	4 (7.5%)	2 (3.8%)

Supplementary Table 1: Description of the experimental study conducted in Diederich et al. [1]. The experiment consisted of three sessions. Within each session, two different response time limits were included: the first and third blocks with $1s$, and the second and fourth blocks with $3s$. Across the sessions, three different levels of induced need, defined as the minimum points to be obtained during one block of trials, were applied with levels 0, 2500, and 3500 points. Given the percentage of trials that prospect A is chosen by each decision-maker (here we report the average across all decision-makers) in the gain and loss frames, the average framing effect is calculated according to Def. 5. Lastly, we report the no. of subjects that do not exhibit a behaviour consistent with PT.

shot, and 6=long shot, and whose probability is indicated by tally marks. Prospects A and B can be computed as a combination of these outcomes, obtaining in total 36 problems. Since 6 are trivial cases, where A = B and $P_{\text{A}} = P_{\text{B}} = 1/2$, there are $36 - 6 = 30$ cases A $\neq$ B for which P_{A} is predicted: these cases are, e.g., (A =1, B =2), (A =1, B =3), ... (A =6, B =1), ... (A =6, B =5). The experimental results are given in Table 2 in [21] under "standard lotteries" as probabilities of choice by 80 subjects, where A prospects are rows and B prospects are columns. Note the antidiagonal values of probability of choice equal to 0.5 for the cases where prospects are identical (A=B).

The data set DS5 consists of 36 gain cases (as defined in Figure 1 of [21] with positive monetary values) and 36 loss cases (as defined in Figure 1 of [21] but with negative monetary values), for a total of 72 data points (which include $6+6 = 12$ trivial cases for which $P_{\text{A}} = P_{\text{B}} = 0.5$). The computation of utility values for the gain and loss cases with multiple outcomes is based on eqs. (1)-(2) in [19].

DS9, Erev et al. [28] Prospects used in the experiments captured by the datasets DS9 and DS12–DS12m are generated by the same algorithm, following a process explained in [28]. For this study, we extract only the baseline cases, which we describe in this and the next paragraph. The data presented by the authors in the tables in Appendices B,D,G, and I, where obtained for 180 problems of which 30 were repeated. From the 180 problems we select those for which: Lottery Num = 1: maximum two outcomes in each prospect; Corr = 0: no correlation; Amb = 0: no ambiguity in prospect formulation. The observed choice probability P_{B} in the column w/o feedback (i.e., single choice) was used.

DS12 and DS12m, Peterson et al. [29] The data is extracted from the file `c13k_selections.csv` (see github.com/jcpeterson/choices13k for a description of the data given by the authors Peterson et al.).

The data is structured as a table $T(i, j)$ where the row index $i = 1, \dots, 13009$ represents a choice problem (with 13009 being the total number of problems considered), and the column $j = 1, \dots, 16$ indicates a variable in {Problem, Feedback, n, Block, Ha, pHa, La, Hb, pHb, Lb, LotShapeB, LotNumB, Amb, Corr, bRate, bRate_std}. We extract cases with the following criteria:

- $T(i, 2)$ = Feedback = "False" or $T(i, 4)$ = Block = 1: No feedback is given to subjects about the reward they received and missed out on after making their selection (only 2380/13009 problems satisfy this condition);

- $T(i, 12)$ = LotNumB = 1: There are only two outcomes in the gamble prospect B;
- $T(i, 13)$ = Amb = "False": There is no ambiguity in the problem formulation, i.e., the decision-maker received complete information and was able to see the probabilities of the outcomes (see [29] for a detailed definition);
- $T(i, 14)$ = Corr = 0: There is no correlation between the payoffs of the two gambles.

The problem IDs for the selected cases are given in the online repository. Note that there are 13009 problem IDs but 14568 rows, i.e., some problems are repeated with and w/o feedback. Our selection of a given problem ID is always the one w/o feedback. Examples of relatively difficult prospects to comprehend and evaluate are summarised in the Supplementary Information of [29].

3.3 First test of cRUM: variability across decision-makers and framing effect

In this section we describe the results obtained on the datasets DS-FR1 (De Martino et al. [24]) and DS-FR2 (Diederich et al. [1]) of Table 1 in the main manuscript.

DS-FR1 (De Martino et al. [24]) For a thorough discussion on the results for the dataset DS-FR1 we refer the reader to the main manuscript. Briefly, Figure 2 in the main manuscript illustrates how the stochastic model $\beta \sim LN[1.8, 0.5]$ reproduces the observed variability in framing effect (Def. 5), evaluated in terms of Pearson correlation coefficient r (left panel of Fig. 3(a)). Variability w.r.t. PT parameters $(\gamma, \delta, \lambda)$ is highlighted in the right panel of Fig. 3(a). Finally, dependency on percentage of total amount offered in sure prospect (i.e., π in the notation of eq. (9)) and on initial amount (i.e., $\pm Y$ in the notation of eq. (9)) is shown in the left and right panels of Fig. 3(b), respectively.

DS-FR2 (Diederich et al. [1]) For completeness, we decided to test our model for β on a bigger dataset (DS-FR2), both in terms of number of subjects, 53, and set of problems (Supplementary Table 1). The unique problems/trials are described in Supplementary Section 3.2.1. Again, the stochastic model $\beta \sim LN[1.8, 0.5]$ reproduces the observed variability in framing effect well, considering that no calibration is involved.

In addition, we decided to estimate (read also: calibrate) the distribution of β using the following methods (M_1–M_2), which estimate parameters of a log-normal distribution of β across individuals. Diverse methods were considered since, to our knowledge, there is no standard method to calibrate weight parameters on framing effect data. In the following, we denote by $y_{k,t} \in \{0, 1\}$ (0 = choose B, 1 = choose A) the observed responses for each decision-maker $k = 1, \dots, 53$ and problem/trial t.

(M_1) **Input:** Responses $y_{k,t} \in \{0, 1\}$ for each decision-maker k and trial t.

Output: Estimated distribution $\hat{\beta} \sim LN[\hat{\mu}, \hat{\sigma}]$ with mean $E[\hat{\beta}] = \exp\left(\hat{\mu} + \frac{\hat{\sigma}^2}{2}\right)$.

Method: *(Grid search)* A 50×50 grid of values (μ_i, σ_j), where $\mu_i \in [0.1, 3]$ and $\sigma_j \in [0.1, 1.5]$ $(i, j = 1, \dots, 50)$ are linearly spaced with no. of points equal to 50 (s.t. the spacing between the points is $\frac{\max - \min}{49}$, with $[\min, \max]$ indicating lower/upper bounds of μ_i or σ_j), is created to fit the distribution $LN[\mu, \sigma]$ on the observed framing effect. The estimated distribution $\hat{\beta} \sim LN[\hat{\mu}, \hat{\sigma}]$ minimises the MSE between observed and estimated framing effect across all decision-makers k:

$$(\hat{\mu}, \hat{\sigma}) = \underset{\substack{(\mu_i, \sigma_j) \\ i,j=1,\dots,50}}{\arg\min} \; \frac{1}{53} \sum_{k=1}^{53} \left(\text{framing effect}(k) - \text{estimated framing effect}(k) \text{ at } (\mu_i, \sigma_j)\right)^2.$$

(M_2) **Input:** Responses $y_{k,t} \in \{0, 1\}$ for each decision-maker k and trial t.

Output: Estimated distribution $\hat{\beta} \sim LN[\hat{\mu}, \hat{\sigma}]$ with mean $E[\hat{\beta}] = \exp\left(\hat{\mu} + \frac{\hat{\sigma}^2}{2}\right)$.

Method: *(Mixed logit)* We follow the work of Revelt and Train [30], who perform mixed logit estimation to evaluate how the parameter β varies over people by maximum simulated likelihood.

As exact maximum likelihood estimation is not possible, the probability of observing a sequence of choices for each subject is approximated by a summation over randomly chosen values of β, drawn from a $LN[\mu, \sigma]$ distribution for given values of the parameters μ, σ. We consider a total no. of draws $N_{\text{DRAWS}} = 1\,000$. The estimated distribution $\hat{\beta} \sim LN[\hat{\mu}, \hat{\sigma}]$ across subjects is the one that maximises the simulated log-likelihood ℓ:

$$
\begin{aligned}
(\hat{\mu}, \hat{\sigma}) &= \underset{(\mu,\sigma)}{\arg\max}\ \ell(\beta(\mu, \sigma)) \\
\text{where} \quad \ell(\beta(\mu, \sigma)) &= \sum_{k=1}^{53} \log \Big(\sum_{r=1}^{N_{\text{DRAWS}}} \prod_t p_t(\beta_{k,r})^{y_{k,t}} (1 - p_t(\beta_{k,r}))^{1-y_{k,t}} \Big) \\
\text{with} \quad \beta_{k,r} &\sim LN[\mu_{k,r}, \sigma_{k,r}] \\
p_t(\beta_{k,r}) &= \frac{1}{1 + \exp(-\beta_{k,r}(\zeta_{\text{A}t} - \zeta_{\text{B}t}))}.
\end{aligned}
$$

$p_t(\beta_{k,r})$ is the probability that decision-maker k chooses A at trial t; the probability of observing the choices $y_{k,t}$ in all trials t is captured by $\prod_t p_t(\beta_{k,r})^{y_{k,t}} (1 - p_t(\beta_{k,r}))^{1-y_{k,t}}$.

In each method (M_1 and M_2) we partition the data in DS-FR2.#, where DS-FR2.# refers to one of the 7 case studies DS-FR2.1–DS-FR2.7, in training and test data (50%-50% partition). Results are shown in Supplementary Fig. 4.

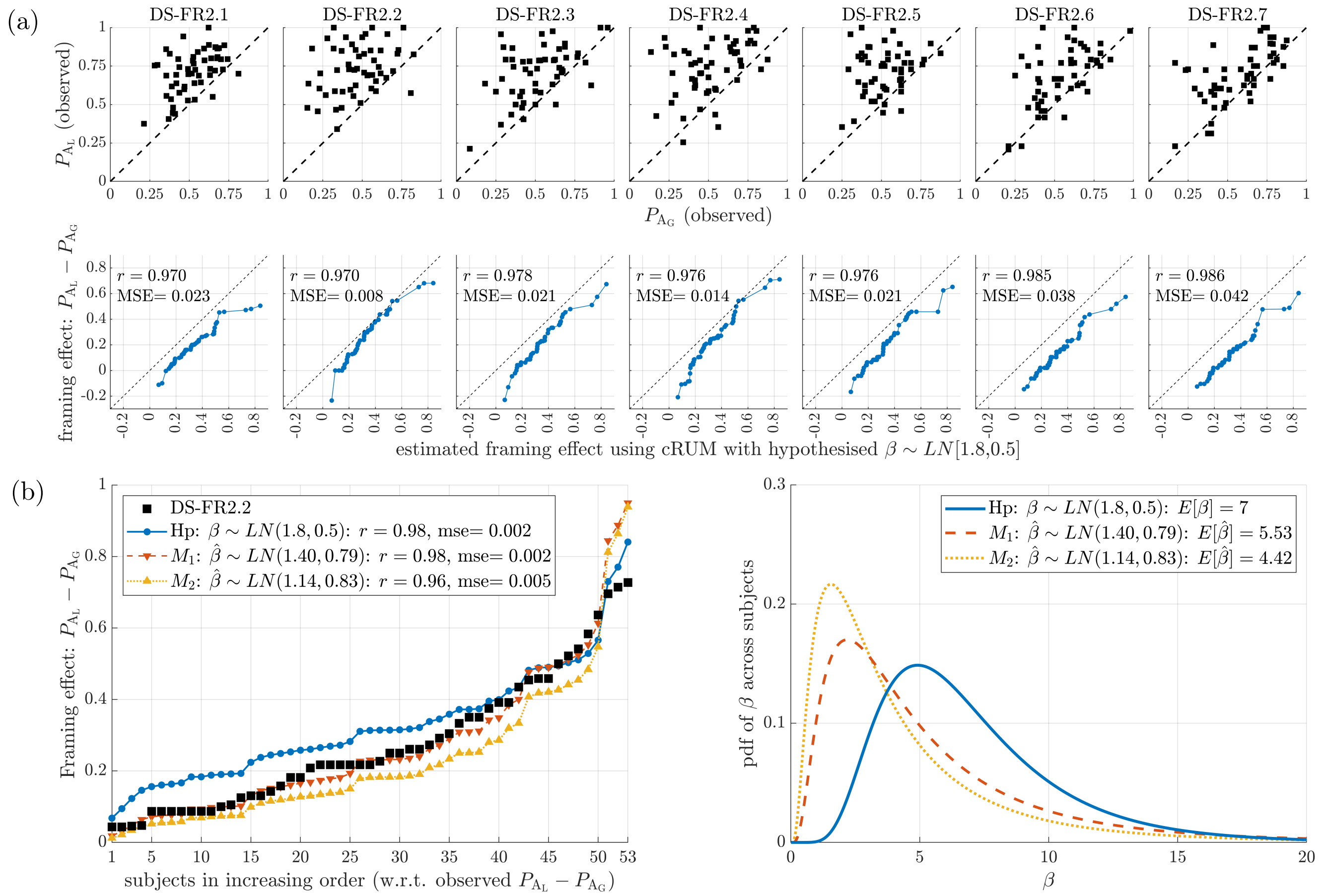
- Training data is used to find the optimal parameters ($\hat{\mu}$, $\hat{\sigma}$) using a 2-fold cross validation procedure:
  - Given N_T the total no. of datapoints in training data, we compute a random partition (D_1, D_2) for 2-fold cross-validation. The partition randomly divides the training data into 2 disjoint folds, each of which has approximately the same number of data points.
  - We train on D_1 and test on D_2; in the test phase we compute MSE between observed and estimated framing effect across all decision-makers using D_2. We repeat the procedure, training on D_2 and testing on D_1.
  - We compare the two values of MSE obtained in the test phase on D_1 and on D_2. We choose the parameters $\hat{\mu}, \hat{\sigma}$, hence stochastic model $\beta \sim LN[\hat{\mu}, \hat{\sigma}]$, that yield lowest MSE.
- Test data is used to evaluate the predictive power of the calibrated stochastic model $\beta \sim LN[\hat{\mu}, \hat{\sigma}]$ and of the proposed (not calibrated) stochastic model $\beta \sim LN[1.8, 0.5]$ (eq. (7)), by computing goodness of fit. Goodness of fit is given in terms of correlation and MSE between observed and estimated framing effect across all decision-makers.

3.4 Second test of cRUM: across decision-makers and datasets

In this section we describe the results obtained on the datasets DS1 to DS12m and aggregated datagroups DGs, DGgl, DGm of Table 1 in the main manuscript. Although we refer the reader to the main manuscript for a thorough discussion on the results, in the following sections we describe the statistical indicators reporting goodness of fit (Supplementary Section 3.4.1) and the performed linear regression analysis between observed and simulated probability of risky choice P_{A} (Supplementary Section 3.4.2).

3.4.1 Evaluation of results and model fit

To check if cRUM is valid for the collected data (DS1 to DS12m), we computed three different statistical measures, namely, Pearson's correlation coefficient (r), mean squared error (MSE), and p-value for the t-test (i.e., for the null hypothesis that the difference between modelled and observed data comes from a normal distribution with mean equal to zero and unknown variance). The fit was computed between the observed values reported in the cited references in Table 1 of the main manuscript and the estimated values using cRUM (eq. (3)) of choice probability P_{A} for prospect A. The higher the value for r (which ranges between -1 and 1) and the value for p-value (which ranges between 0 and 1), and the smaller



Supplementary Fig. 4: (a): Testing cRUM on the dataset DS-FR2 and its case studies DS-FR2.# (Supplementary Table 1 for details). The top panels show the observed probabilities of risky choice in loss (P_{A_L}) and gain (P_{A_G}) frames for each decision-maker; according to Remark 1, the data are consistent with PT if $P_{A_L} > 0.5 > P_{A_G}$ (area above dashed black line). The bottom panels illustrate observed vs estimated framing effect using cRUM with $\beta \sim LN[1.8, 0.5]$ (eq. (7)), and associated values of correlation r and MSE. (b): Comparing hypothesised (eq. (7)) vs calibrated distributions of the choice parameter β for DS-FR2.2, which is the case study corresponding to lowest no. of decision-makers exhibiting a nonpositive framing effect (Supplementary Table 1). The left panel shows observed and estimated framing effect using test data. The right panel shows the distribution of β. The methods M_1, M_2 used to estimate the distribution of β are described in Supplementary Section 3.3.

the value for MSE (which is greater than 0), the "stronger" the evidence that our model indeed is a good population-representative model.

Supplementary Fig. 5(a), Supplementary Fig. 6(a), and Fig. 4(a) illustrate the statistical indicators for variations of PT parameters illustrated in Table 2(b) in the manuscript, namely, EU PT parameters, standard PT parameters, and PT parameters as in Table 1, respectively.

3.4.2 Linear regression plots

We performed a linear regression analysis first on the aggregated data groups listed in Table 2(a), and then on the datasets listed in Table 1 in the main manuscript.

Supplementary Fig. 7(a), Fig. 5(b), and Supplementary Fig. 7(b) illustrate the observed choice probability vs estimated choice probability P_A using cRUM with $E[\beta] = 4, 7, 10$, respectively, and the corresponding linear regression lines, for all datagroups of Table 2(a) and all PT parameters scenarios of Table 2(b). The same correlation analysis is plotted in Fig. 6, Supplementary Fig. 8, Supplementary Fig. 9, and Supplementary Fig. 10 for each dataset of Table 1, $E[\beta] = 4, 7, 10$, and all PT parameters scenarios of Table 2(b), namely, EU PT parameters (Supplementary Fig. 10), standard PT parameters (Supplementary Fig. 9), and PT parameters from Table 1 (Fig. 6 and Supplementary Fig. 8).

Note that the choice to consider $E[\beta] = 4, 7, 10$ is inspired by the fact that the range $[4, 10]$ approximates the 68% confidence interval of the proposed stochastic model for β (Supplementary Fig. 2).

References

[1] Diederich, A., Wyszynski, M. & Traub, S. Need, frames, and time constraints in risky decision-making. *Theory and Decision* **89**, 1–37 (2020).

[2] von Neumann, J. & Morgenstern, O. *Theory of Games and Economic Behavior* (Princeton University Press, 1944).

[3] Bernoulli, D. Exposition of a new theory on the measurement of risk. *Econometrica* **22**, 23–36 (1953).

[4] McFadden, D. L. Quantal choice analysis: a survey. *Annals of Economic and Social Measurement* **5**, 475–492 (1976).

[5] Luce, R. D. The choice axiom after twenty years. *Journal of Mathematical Psychology* **15**, 215–233 (1977).

[6] Yellott, J. I. J. The relationship between Luce's choice axiom, Thurstone's theory of comparative judgement, and the double exponential distribution. *Journal of Mathematical Psychology* **15**, 109–144 (1977).

[7] Train, K. E. *Discrete Choice Methods with Simulation* (Cambridge University Press, Cambridge, 2003), 2nd edn.

[8] Thurstone, L. L. A law of comparative judgment. *Psychological Review* **34**, 273–286 (1927).

[9] Train, K. E. Mixed logit with a flexible mixing distribution. *Journal of Choice Modelling* **19**, 40–53 (2016).

[10] Rieskamp, J. The probabilistic nature of preferential choice. *Journal of Experimental Psychology: Learning Memory and Cognition* **34**, 1446–1465 (2008).

[11] Wang, Z. J. & Busemeyer, J. R. *Cognitive Choice Modeling* (The MIT Press, 2021).

[12] Glickman, M. *et al.* The formation of preference in risky choice. *PLoS Computational Biology* **15**, 1–25 (2019).

[13] McKelvey, R. D. & Palfrey, T. R. Quantal response equilibria for normal form games. *Games and Economic Behavior* **10**, 6–38 (1995).

[14] Rosenfeld, A. & Kraus, S. Predicting human decision-making from prediction to action. *Synthesis Lectures on Artificial Intelligence and Machine Learning* **12**, 1–149 (2018).

[15] Erev, I., Roth, A. E., Slonim, R. L. & Barron, G. Predictive value and the usefulness of game theoretic models. *International Journal of Forecasting* **18**, 359–368 (2002).

[16] Ajzen, I. The theory of planned behavior. *Organizational Behavior and Human Decision Processes* **50**, 179–211 (1991).

[17] Ricci, M. & Gallistel, R. Accurate step-hold tracking of smoothly varying periodic and aperiodic probability. *Attention, Perception, and Psychophysics* **79**, 1480–1494 (2017).

[18] Gallistel, C. R., Krishan, M., Liu, Y., Miller, R. & Latham, P. E. The perception of probability. *Psychological Review* **121**, 96–123 (2014).

[19] Tversky, A. & Kahneman, D. Advances in prospect theory: cumulative representation of uncertainty. *Journal of Risk and Uncertainty* **5**, 297–323 (1992).

[20] Stott, H. P. Cumulative prospect theory's functional menagerie. *Journal of Risk and Uncertainty* **32**, 101–130 (2006).

[21] Lopes, L. L. & Oden, G. C. The role of aspiration level in risky choice: a comparison of cumulative prospect theory and SP/A theory. *Journal of Mathematical Psychology* **43**, 286–313 (1999).

[22] Tversky, A. & Kahneman, D. The framing of decisions and the psychology of choice. *Science* **211**, 453–458 (1981).

[23] Kühberger, A. The influence of framing on risky decisions: a meta-analysis. *Organizational Behavior and Human Decision Processes* **75**, 23–55 (1998).

[24] De Martino, B., Kumaran, D., Seymour, B. & Dolan, R. J. Frames, biases, and rational decision-making in the human brain. *Science* **313**, 684–687 (2006).

[25] Steiger, A. & Kühberger, A. A Meta-Analytic Re-Appraisal of the Framing Effect. *Zeitschrift für Psychologie* **226**, 45–55 (2018).

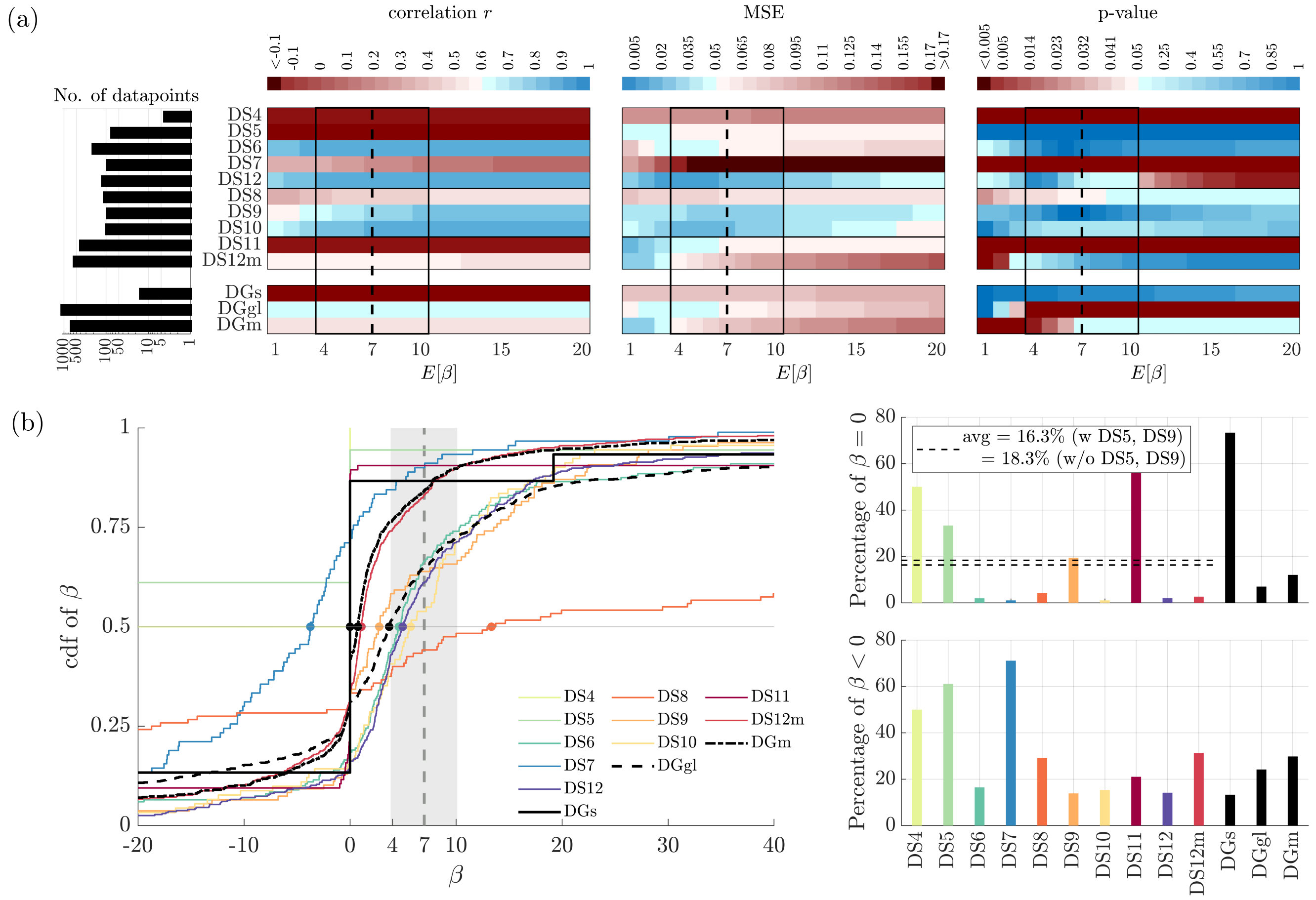
[26] Nabi, R. L. *et al.* Can emotions capture the elusive gain-loss framing effect? A meta-analysis. *Communication Research* **47**, 1107–1130 (2020).

[27] McDonald, K., Graves, R., Yin, S., Weese, T. & Sinnott-Armstrong, W. Valence framing effects on moral judgments: A meta-analysis. *Cognition* **212**, 104703 (2021).

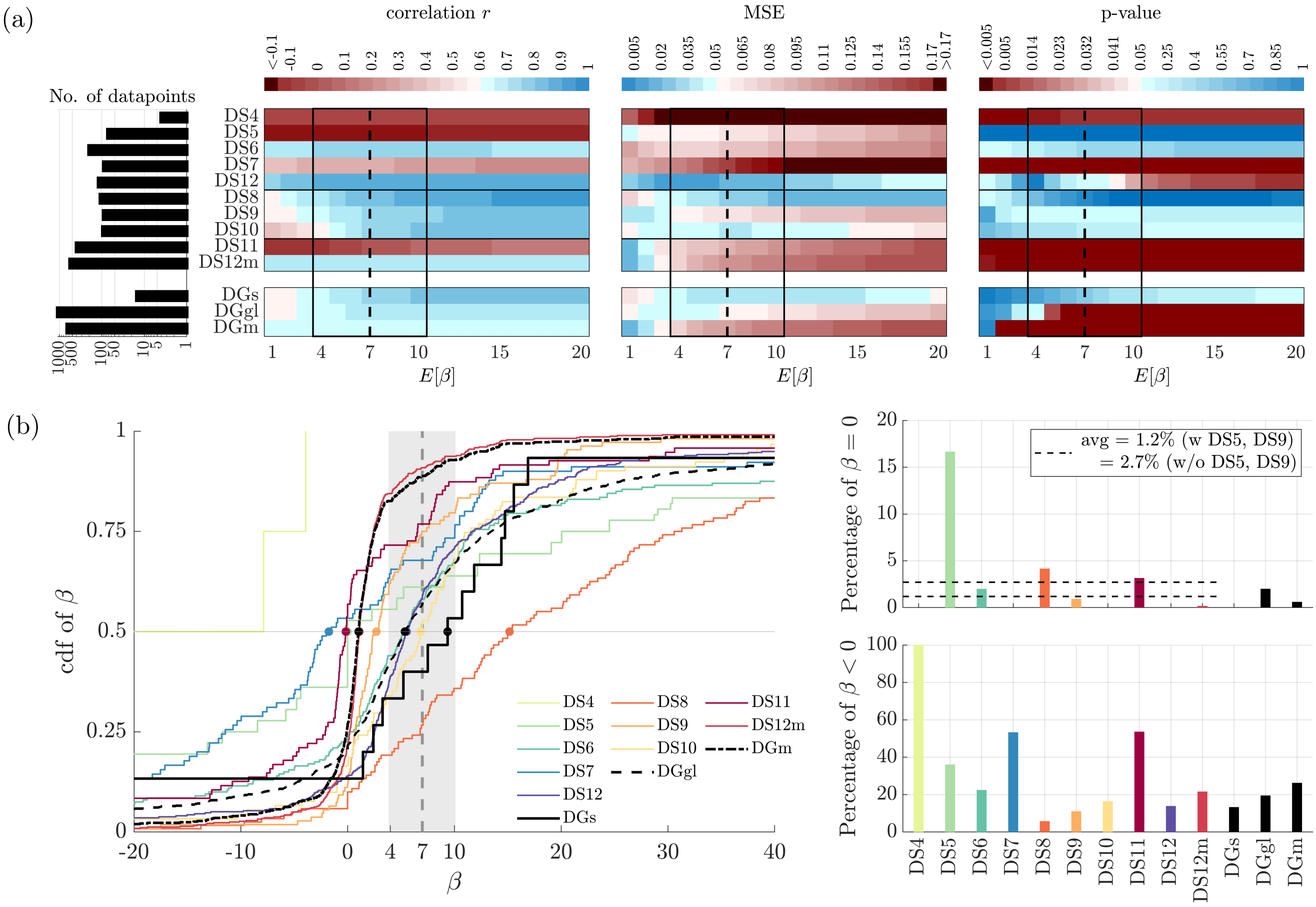
[28] Erev, I., Ert, E., Plonsky, O., Cohen, D. & Cohen, O. From anomalies to forecasts: toward a descriptive model of decisions under risk, under ambiguity, and from experience. *Psychological Review* **124**, 369–409 (2017).

[29] Peterson, J. C., Bourgin, D. D., Agrawal, M., Reichman, D. & Griffiths, T. L. Using large-scale experiments and machine learning to discover theories of human decision-making. *Science* **372**, 1209–1214 (2021).

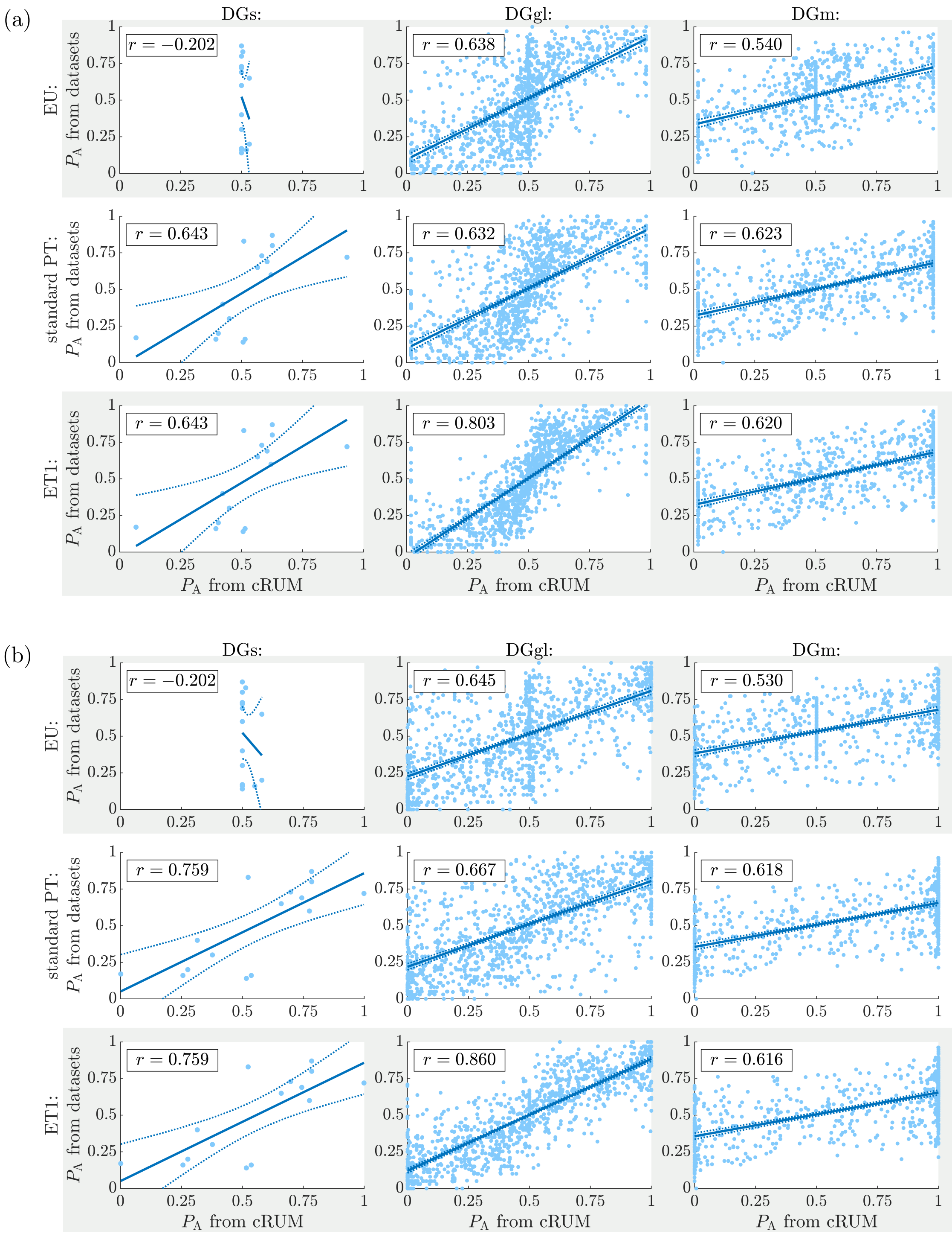
[30] Revelt, D. & Train, K. E. Mixed logit with repeated choices: households' choices of appliance efficiency level. *The Review of Economics and Statistics* **80**, 647–657 (1998).



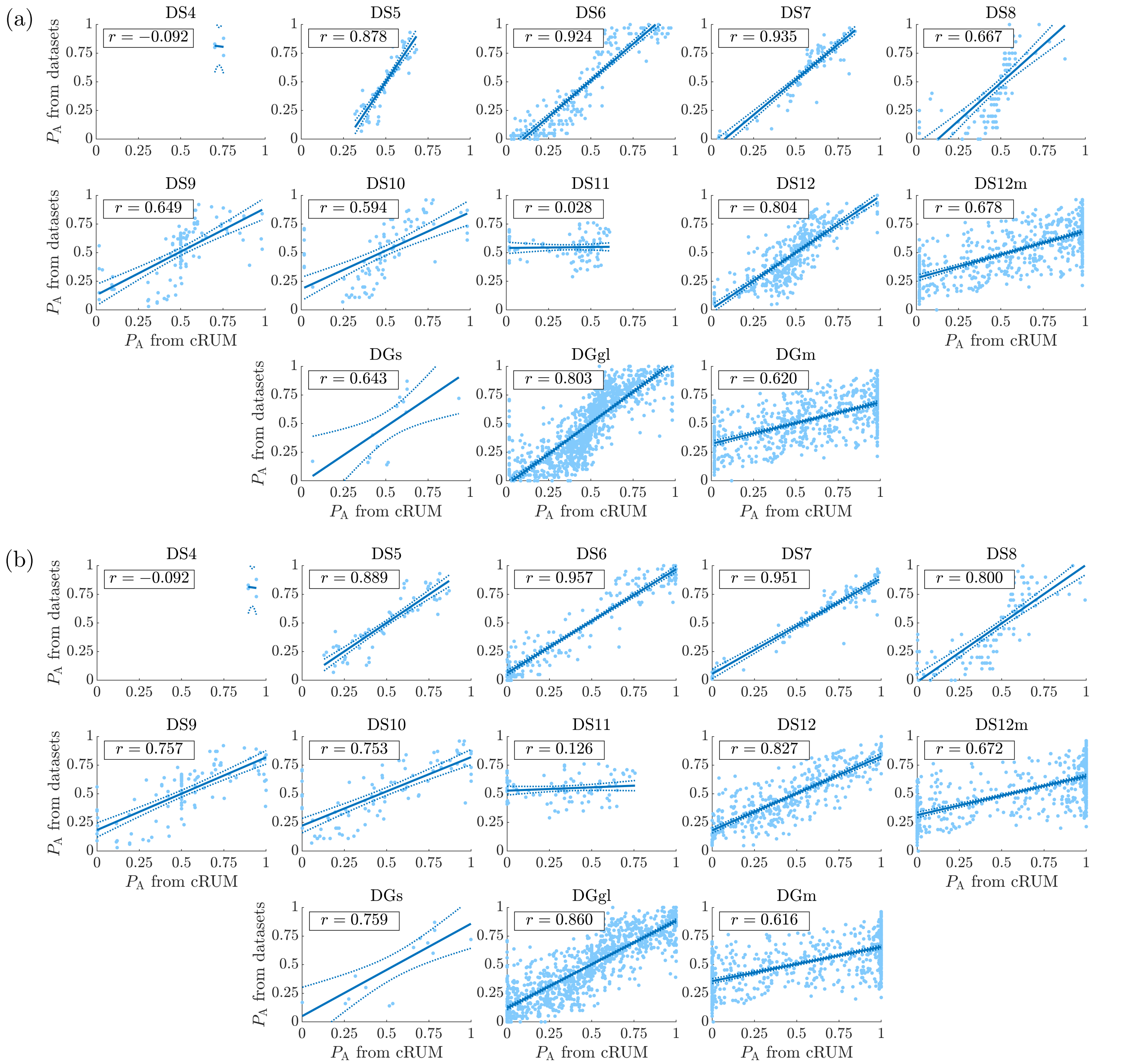
Supplementary Fig. 5: EU PT parameters (Table 2(b), to compare with Fig. 4(a). (a): Model fit in terms of statistical measures (correlation r, MSE, p-value of t-test) between observed choice probability P_A and predicted choice probability P_A using cRUM with $E[\beta] \in \{1, 2, \ldots, 20\}$. Blue colour signals what we consider desirable (high correlation, low MSE, and high p-value). (b): cdf of β obtained from eq. (9) in the main manuscript (left), percentage of datapoints for which $\beta = 0$ (top right), and percentage of datapoints for which $\beta < 0$ (bottom right) for all datasets (colour-coded) and datagroups (black) of Tables 1 and 2(a). The proposed ambiguity indicators are the percentage of zero and negative β values (right panels), and the β medians (indicated by dot symbols in the curves, left panel).



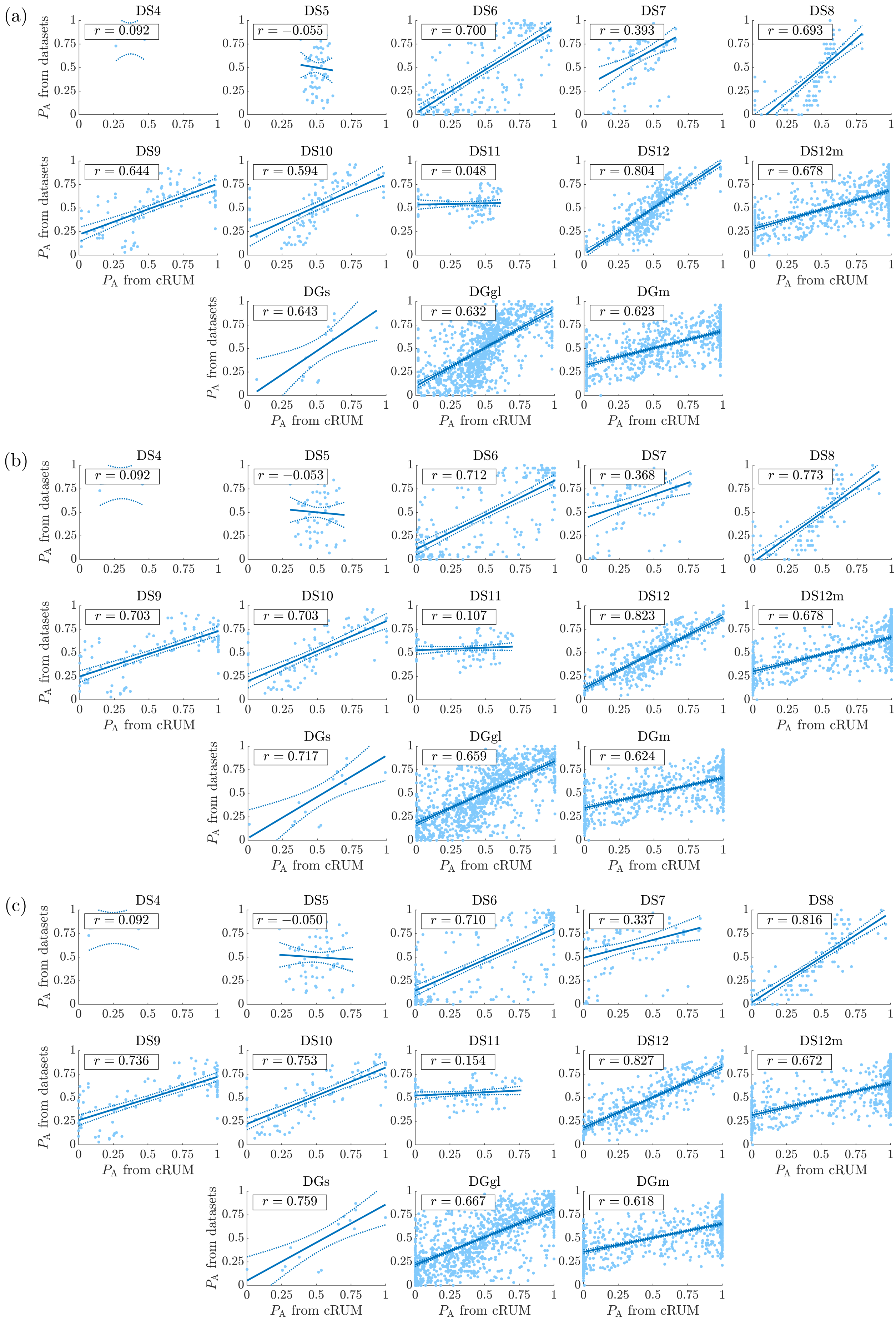
Supplementary Fig. 6: Standard PT parameters (Table 2(b)), to compare with Fig. 4(a). (a): Model fit in terms of statistical measures (correlation r, MSE, p-value of t-test) between observed choice probability P_A and predicted choice probability P_A using cRUM with $E[\beta] \in \{1, 2, \ldots, 20\}$. Blue colour signals what we consider desirable (high correlation, low MSE, and high p-value). (b): cdf of β obtained from eq. (9) in the main manuscript (left), percentage of datapoints for which $\beta = 0$ (top right), and percentage of datapoints for which $\beta < 0$ (bottom right) for all datasets (colour-coded) and datagroups (black) of Tables 1 and 2(a). The proposed ambiguity indicators are the percentage of zero and negative β values (right panels), and the β medians (indicated by dot symbols in the curves, left panel).



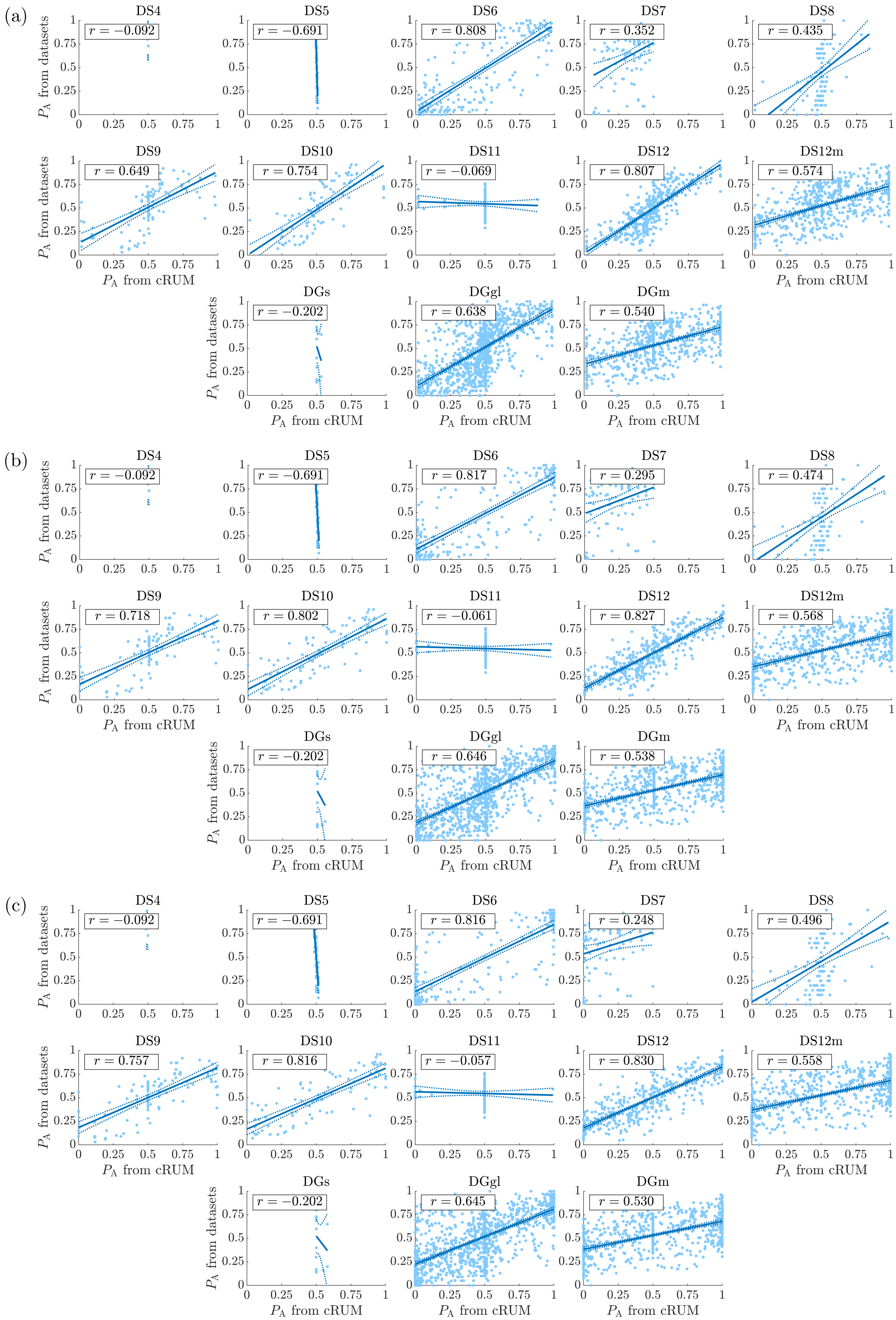
Supplementary Fig. 7: Observed choice probability vs estimated choice probability P_A using cRUM and corresponding linear regression line, for all PT parameters scenarios of Table 2(b) and all datagroups of Table 2(a). (a): cRUM with $E[\beta] = 4$; (b): cRUM with $E[\beta] = 10$. To compare with Fig. 5(b) for cRUM with $E[\beta] = 7$.



Supplementary Fig. 8: Observed choice probability vs estimated choice probability P_A using cRUM and corresponding linear regression line, for PT parameters from Table 1 and all datasets of Table 1. (a): cRUM with $E[\beta] = 4$; (b): cRUM with $E[\beta] = 10$. To compare with Fig. 6 for cRUM with $E[\beta] = 7$.



Supplementary Fig. 9: Observed choice probability vs estimated choice probability P_{A} using cRUM and corresponding linear regression line, for standard PT parameters (Table 2(b)) and all datasets of Table 1. (a): cRUM with $E[\beta] = 4$; (b): cRUM with $E[\beta] = 7$; (c): cRUM with $E[\beta] = 10$.



Supplementary Fig. 10: Observed choice probability vs estimated choice probability P_{A} using cRUM and corresponding linear regression line, for EU PT parameters (Table 2(b)) and all datasets of Table 1. (a): cRUM with $E[\beta] = 4$; (b): cRUM with $E[\beta] = 7$; (c): cRUM with $E[\beta] = 10$.